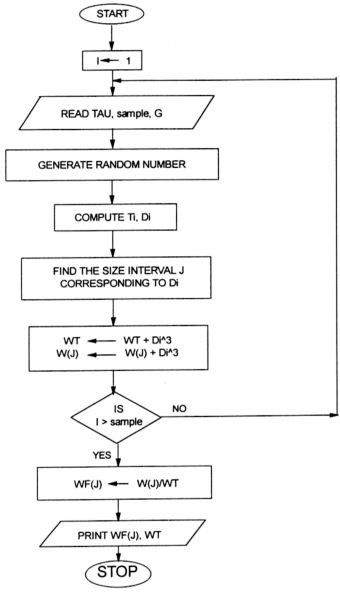
APPENDIX I

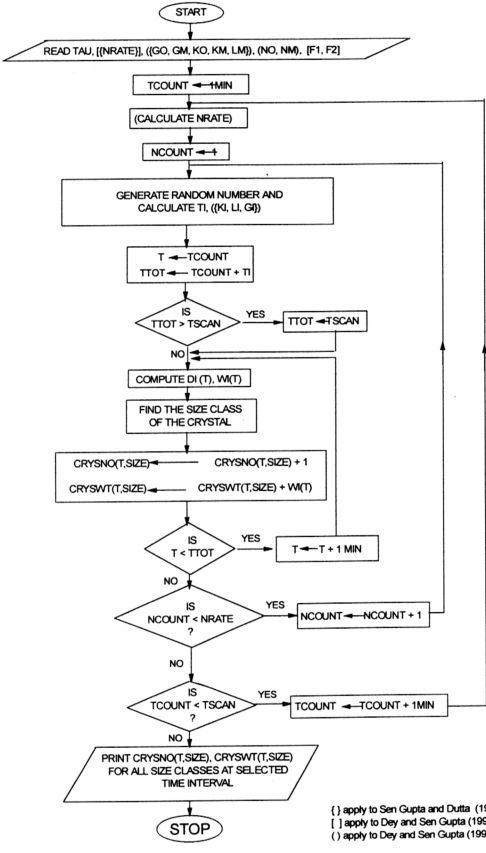
Appendix I.1

Outline of Simulation Algorithm for Steady State



Appendix I.2

Outline of Simulation Algorithm for Transient CSD



{ } apply to Sen Gupta and Dutta (1990c)
[ ] apply to Dey and Sen Gupta (1993)
( ) apply to Dey and Sen Gupta (1993a)

Appendix I.3

The Detailed Procedure of Generating Random Samples of Growth Rate and Shape Factor from the Normal Density Function

The normal distribution is a continuous distribution completely defined by its mean m and standard deviation σ. The density function of a normal distribution (m, σ) is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-(x-m)^2 / 2\sigma^2] \qquad \text{(AI.3.1)}$$

It is assumed that the dispersion range is given by $m \pm 3\sigma$. The dispersion band spread over the 3σ limits of the mean value guarantees 99.7% occurrence of the random variable within this region. For example, in case of growth rate dispersion,

$$G_O = m - 3\sigma \qquad \text{(AI.3.2)}$$

$$G_M = m + 3\sigma \qquad \text{(AI.3.3)}$$

Hence, $$m = (G_M + G_O)/2 \qquad \text{(AI.3.4)}$$

and $$\sigma = (G_M - G_O)/6 \qquad \text{(AI.3.5)}$$

In order to generate random samples, the following steps are followed:

$$RND = \exp[-(x-m)^2 / 2\sigma^2] \tag{AI.3.6}$$

which satisfies the condition that,

$$f(x) = 0 \text{ when } RND = 0$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \text{ when } RND = 1$$

From Eq. (AI.3.6), one can write

$$x = m \pm \sqrt{-2\sigma^2 \log(RND)} \tag{AI.3.7}$$

To select a single value of x from Eq. (AI.3.7), a random number is generated. If the random number is smaller than 0.5, the discriminate is assigned a negative sign, otherwise it is taken as positive. Thus the growth rate and shape factor of any crystal can be found as Eq. (1.65) and Eq. (1.66) when they are normally distributed over a known range.

Appendix I.4

Derivation of Eqs (1.92), (1.93) and (1.94)

The probability that a crystal will be in state j at time $(t+\Delta t)$ arises from two independent sequential events; one is that the crystal is in state $(j-1)$ at time t and will be in state j at time $(t+\Delta t)$, and the other is that it is in state j at time t and will remain in this state at time $(t+\Delta t)$. According to the assumptions and definitions given at the model elaboration of section 1.3.2.1, the conditional probability that the crystal will be in state j at time $(t+\Delta t)$ given that it is in state $(j-1)$ at time t can be approximated by

$$\lambda_{j-1}(t)\Delta t + o(\Delta t), \quad \text{for } j>1 \tag{AI.4.1}$$

Then the probability of the first sequential events is

$$P_{j-1}(t)\left[\lambda_{j-1}(t)\Delta t + o(\Delta t)\right] \tag{AI.4.2}$$

Similarly, the probability of the second sequential events is

$$P_j(t)\left[1-\lambda_j(t)\Delta t + o(\Delta t)\right] \tag{AI.4.3}$$

Hence, the probability that the crystal will be in state j at time $(t+\Delta t)$ is the sum of the probabilities of these two exclusive sequential events, i.e.,

$$P_j(t+\Delta t) = P_{j-1}(t)\left[\lambda_{j-1}(t)\Delta t + o(\Delta t)\right] + P_j(t)\left[1-\lambda_j(t)\Delta t + o(\Delta t)\right] \qquad \text{(AI.4.4)}$$

By rearranging this expression, we have,

$$\frac{P_j(t+\Delta t) - P_j(t)}{\Delta t} = \lambda_{j-1}(t)P_{j-1}(t) - \lambda_j(t)P_j(t) + \frac{o(\Delta t)}{\Delta t}\left[P_{j-1}(t) + P_j(t)\right] \qquad \text{(AI.4.5)}$$

Taking the limit as $\Delta t \to 0$ yields

$$\frac{dP_j(t)}{dt} = \lambda_{j-1}(t)P_{j-1}(t) - \lambda_j(t)P_j(t) \qquad \text{(AI.4.6)}$$

The probability that a crystal will be in state 1 at time $(t+\Delta t)$ only arises from the sequential events that it is in state 1 at time t and will remain in state 1 at time $(t+\Delta t)$. As a result, we have

$$\frac{dP_1(t)}{dt} = -\lambda_1(t)P_1(t) \qquad \text{(AI.4.7)}$$

From Eq. (AI.4.6), we have, for the final state, i.e., $j = M$,

$$\frac{dP_M(t)}{dt} = \lambda_{M-1}(t)P_{M-1}(t) \tag{AI.4.8}$$

Eqs (AI.4.6), (AI.4.7) and (AI.4.8) are Eqs (1.92), (1.93) and (1.94) respectively.

$$\frac{dP_M(t)}{dt} = \lambda_{M-1}(t)P_{M-1}(t) \qquad \text{(AI.4.8)}$$

Eqs (AI.4.6), (AI.4.7) and (AI.4.8) are Eqs (1.92), (1.93) and (1.94) respectively.

Appendix I.5

Multinomial Distribution

Random vector $(X_1, X_2, \ldots, X_M)$ is multinomially distributed with parameters $(n, P_1, P_2, \ldots, P_M)$ if the joint probability mass function of $(x_1, x_2, \ldots, x_M)$ is given by

$$P[X_1 = x_1, X_2 = x_2, \ldots, X_M = x_M] = \begin{cases} \dfrac{n!}{\prod_{j=1}^{M} x_j!} \prod_{j=1}^{M} P_j^{x_j} & \text{if } \sum_{j=1}^{M} x_j = n \\ 0 & \text{otherwise} \end{cases} \quad \text{(AI.5.1)}$$

where x_i, $i = 1, 2, \ldots, M$ are nonnegative integers and P_j denotes the probability that X_j takes the value of x_j.

For the present case, a seed (i.e., a crystal in state 1 at time 0) will be in state j at time t with a probability of $P_j(t)$; therefor, with $m(0)$ seeds present initially, the joint probability mass function of $(n_1, n_2, \ldots, n_M)$ becomes

$$P[N_1(t) = n_1, N_2(t) = n_2, \ldots, N_M(t) = n_M] = \begin{cases} \dfrac{m(0)!}{\prod_{j=1}^{M} n_j!} \prod_{j=1}^{M} [P_j(t)]^{n_j} & \text{if } \sum_{j=1}^{M} n_j = m(0) \\ 0 & \text{otherwise} \end{cases} \quad \text{(AI.5.2)}$$

For any fixed value of j, $j = 1, 2, \ldots, M$, a crystal in state 1 at time 0 will be either in state j at time t with a probability of $P_j(t)$ or not in state j at time t with a probability of $\left[1 - P_j(t)\right]$. Consequently, out of $m(0)$ seeds, the probability that n_j of them will be in state j at time t is

$$\binom{m(0)}{n_j} P_j(t)^{n_j} \left[1 - P_j(t)\right]^{m(0)-n_j} \tag{AI.5.3}$$

which is a binomial distribution. It is well known that the mean and variance of such a distribution are,

$$E[N_j(t)] = m(0)P_j(t) \tag{AI.5.4}$$

and

$$Var[N_j(t)] = m(0)P_j(t)[1 - P_j(t)] \tag{AI.5.5}$$

respectively.

Appendix I.6

Derivation of Eqs (1.129) and (1.130)

For convenience, Eq. (1.116) is reiterated below:

$$\frac{dP(j,t)}{dt} = \left(\mathrm{E}^{-1} - 1\right)\alpha^{j-k} P(j,t), \qquad j = 1, 2, \ldots, M \tag{AI.6.1}$$

Taking the transformation

$$j = \Omega\phi(t) + \Omega^{1/2} x \tag{AI.6.2}$$

and collecting terms of the order $\Omega^{1/2}$ and Ω^{0} yield, respectively (VAN KAMPEN, 1992),

$$\frac{d\phi}{dt} = \mathrm{f}_1(\phi) \tag{AI.6.3}$$

and

$$\frac{\partial q}{\partial t} = -\mathrm{f}_2(\phi)\frac{\partial (xq)}{\partial x} + \mathrm{f}_3(\phi)\frac{\partial^2 q}{\partial x^2} \tag{AI.6.4}$$

Eq. (AI.6.3) is the macroscopic equation and Eq. (AI.6.4) is a linear Fokker-Plank equation (VAN KAMPEN, 1976, 1992). The first and second moments of the random

variable X are governed by the coefficients of the partial derivatives on the right-hand side of the latter through the following two expressions.

$$\frac{d\langle x\rangle}{dt} = f_2(\phi)\langle x\rangle \tag{AI.6.5}$$

and

$$\frac{d\langle x^2\rangle}{dt} = -2f_2(\phi)\langle x^2\rangle + 2f_3(\phi) \tag{AI.6.6}$$

A comparison of Eq. (1.128) in the text with Eq. (AI.6.4) yields

$$f_2(\phi) = a_1\phi^{-(k+1)}\frac{k}{\phi} \tag{AI.6.7}$$

and

$$f_3(\phi) = \frac{a_1\phi^{-k}}{2} \tag{AI.6.8}$$

As a result, Eqs. (1.129) and (1.130) can be recovered from Eqs. (AI.6.5) and (AI.6.6) respectively.

APPENDIX II

Appendix II.1

Sample Program and Results for Transient CSD for the Size-Dependent Growth by Employing the ASL Model

Declaration

```
Dim wtfr(0 To 16) As Single
Dim cmwtfr(0 To 16) As Single
Dim wet(0 To 16), cry(0 To 16) As Single
Dim cr(1 To 780, 0 To 16) As Single
Dim wt(1 To 780, 0 To 16)  As Single
Dim gzero, nt, nmin, nmax, size, cryswt, v As Single
Dim t1, t2, times, tau, ttot, tmax, b, crysno As Single
Dim k As Integer
```

Main Program

```
Sub main ()

tau = Val(asl.Text2.Text)
gzero = Val(asl.Text3.Text)
b = Val(asl.Text4.Text)
nt = 1000

Open "m_asl" For Output As #1
Print #1, "Results of transient MC Simulation for Size Dependent Growth"
Print #1, "Average residence time=" & tau; " seconds"
Print #1, "Growth rate of crystals at zero size=" & gzero; "mm/s"
Print #1, "ASL parameter=" & b
Print #1, "Tau=" & tau
Print #1, "Nucleation rate=" & nt; "nuclei/min"

tmax = 13 * (tau / 60)
For t1 = 1 To tmax
  Randomize
  For n = 1 To nt
    t2 = -(tau / 60) * Log(Rnd)
    ttot = t1 + t2
    If ttot > tmax Then ttot = tmax
    For times = t1 To ttot Step 1
      v = (1 + (((times - t1) * (1 - b)) / (tau / 60))) ^ (1 / (1 - b))
```

```
        size = (gzero * tau * (v - 1))
        If size > .074 Then
          GoTo 250
        Else
          k = 0
          GoTo 410
        End If
250     If size > .163 Then
          GoTo 260
        Else
          k = 1
          GoTo 410
        End If
260     If size > .2 Then
          GoTo 270
        Else
          k = 2
          GoTo 410
        End If
270     If size > .231 Then
          GoTo 280
        Else
          k = 3
          GoTo 410
        End If
280     If size > .281 Then
          GoTo 290
        Else
          k = 4
          GoTo 410
        End If
290     If size > .325 Then
          GoTo 300
        Else
          k = 5
          GoTo 410
        End If
300     If size > .4 Then
          GoTo 310
        Else
          k = 6
          GoTo 410
        End If
310     If size > .47 Then
          GoTo 320
        Else
```

```
        k = 7
        GoTo 410
      End If
320   If size > .555 Then
        GoTo 330
      Else
        k = 8
        GoTo 410
      End If
330   If size > .655 Then
        GoTo 340
      Else
        k = 9
        GoTo 410
      End If
340   If size > .785 Then
        GoTo 350
      Else
        k = 10
        GoTo 410
      End If
350   If size > .924 Then
        GoTo 360
      Else
        k = 11
        GoTo 410
      End If
360   If size > 1.072 Then
        GoTo 370
      Else
        k = 12
        GoTo 410
      End If
370   If size > 1.268 Then
        GoTo 380
      Else
        k = 13
        GoTo 410
      End If
380   If size > 1.548 Then
        GoTo 390
      Else
        k = 14
        GoTo 410
      End If
390   If size > 1.848 Then
```

```
            GoTo 400
          Else
            k = 15
            GoTo 410
          End If
400       k = 16
410       cr(times, k) = cr(times, k) + 1
          wt(times, k) = wt(times, k) + (size ^ 3)
        Next
      Next
    Next
    For times = (tau / 60) To tmax Step .5 * (tau / 60)
      For k = 0 To 16
        wet(k) = wt(times, k)
        cryswt = cryswt + wet(k)
        cry(k) = cr(times, k)
        crysno = crysno + cry(k)
      Next
      wtfr(0) = wet(0) / cryswt
      cmwtfr(0) = wtfr(0)
      For k = 1 To 16
        wtfr(k) = wet(k) / cryswt
        cmwtfr(k) = cmwtfr(k - 1) + wtfr(k)
      Next
      Print #1, "time=" & times; "min"
      Print #1, "Total number of crystals=" & crysno
      For k = 0 To 16
        Print #1, "MASS UNDER-SIZE IN CLASS("; k; ")="; cmwtfr(k)
        Print #1, "NO. OF CRYSTALS IN CLASS("; k; ")="; cr(times, k)
      Next
      cryswt = 0
      crysno = 0
.Next
 End Sub
```

Sample of results (for Experiment No. 1)

Results of transient MC Simulation for Size Dependent Growth
Average residence time=965 seconds
Growth rate of crystals at zero size=.000026mm/s
ASL parameter=.566
Tau=965
Nucleation rate=1000nuclei/min

time=24.125min
Total number of crystals=12888
MASS UNDER-SIZE IN CLASS(0)= 1
NO. OF CRYSTALS IN CLASS(0)= 12888
MASS UNDER-SIZE IN CLASS(1)= 1
NO. OF CRYSTALS IN CLASS(1)= 0
MASS UNDER-SIZE IN CLASS(2)= 1
NO. OF CRYSTALS IN CLASS(2)= 0
MASS UNDER-SIZE IN CLASS(3)= 1
NO. OF CRYSTALS IN CLASS(3)= 0
MASS UNDER-SIZE IN CLASS(4)= 1
NO. OF CRYSTALS IN CLASS(4)= 0
MASS UNDER-SIZE IN CLASS(5)= 1
NO. OF CRYSTALS IN CLASS(5)= 0
MASS UNDER-SIZE IN CLASS(6)= 1
NO. OF CRYSTALS IN CLASS(6)= 0
MASS UNDER-SIZE IN CLASS(7)= 1
NO. OF CRYSTALS IN CLASS(7)= 0
MASS UNDER-SIZE IN CLASS(8)= 1
NO. OF CRYSTALS IN CLASS(8)= 0
MASS UNDER-SIZE IN CLASS(9)= 1
NO. OF CRYSTALS IN CLASS(9)= 0
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 0
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=40.2083333333333min
Total number of crystals=15175
MASS UNDER-SIZE IN CLASS(0)= .404131
NO. OF CRYSTALS IN CLASS(0)= 14107
MASS UNDER-SIZE IN CLASS(1)= 1
NO. OF CRYSTALS IN CLASS(1)= 1068
MASS UNDER-SIZE IN CLASS(2)= 1
NO. OF CRYSTALS IN CLASS(2)= 0
MASS UNDER-SIZE IN CLASS(3)= 1

NO. OF CRYSTALS IN CLASS(3)= 0
MASS UNDER-SIZE IN CLASS(4)= 1
NO. OF CRYSTALS IN CLASS(4)= 0
MASS UNDER-SIZE IN CLASS(5)= 1
NO. OF CRYSTALS IN CLASS(5)= 0
MASS UNDER-SIZE IN CLASS(6)= 1
NO. OF CRYSTALS IN CLASS(6)= 0
MASS UNDER-SIZE IN CLASS(7)= 1
NO. OF CRYSTALS IN CLASS(7)= 0
MASS UNDER-SIZE IN CLASS(8)= 1
NO. OF CRYSTALS IN CLASS(8)= 0
MASS UNDER-SIZE IN CLASS(9)= 1
NO. OF CRYSTALS IN CLASS(9)= 0
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 0
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=56.29166666666667min
Total number of crystals=15978
MASS UNDER-SIZE IN CLASS(0)= .1531241
NO. OF CRYSTALS IN CLASS(0)= 14105
MASS UNDER-SIZE IN CLASS(1)= .8112866
NO. OF CRYSTALS IN CLASS(1)= 1739
MASS UNDER-SIZE IN CLASS(2)= 1
NO. OF CRYSTALS IN CLASS(2)= 134
MASS UNDER-SIZE IN CLASS(3)= 1
NO. OF CRYSTALS IN CLASS(3)= 0
MASS UNDER-SIZE IN CLASS(4)= 1
NO. OF CRYSTALS IN CLASS(4)= 0
MASS UNDER-SIZE IN CLASS(5)= 1
NO. OF CRYSTALS IN CLASS(5)= 0
MASS UNDER-SIZE IN CLASS(6)= 1
NO. OF CRYSTALS IN CLASS(6)= 0
MASS UNDER-SIZE IN CLASS(7)= 1
NO. OF CRYSTALS IN CLASS(7)= 0
MASS UNDER-SIZE IN CLASS(8)= 1
NO. OF CRYSTALS IN CLASS(8)= 0
MASS UNDER-SIZE IN CLASS(9)= 1
NO. OF CRYSTALS IN CLASS(9)= 0
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 0
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0

MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=72.375min
Total number of crystals=16256
MASS UNDER-SIZE IN CLASS(0)= 7.836225E-02
NO. OF CRYSTALS IN CLASS(0)= 14058
MASS UNDER-SIZE IN CLASS(1)= .4298482
NO. OF CRYSTALS IN CLASS(1)= 1739
MASS UNDER-SIZE IN CLASS(2)= .6309605
NO. OF CRYSTALS IN CLASS(2)= 248
MASS UNDER-SIZE IN CLASS(3)= .7962059
NO. OF CRYSTALS IN CLASS(3)= 119
MASS UNDER-SIZE IN CLASS(4)= 1
NO. OF CRYSTALS IN CLASS(4)= 92
MASS UNDER-SIZE IN CLASS(5)= 1
NO. OF CRYSTALS IN CLASS(5)= 0
MASS UNDER-SIZE IN CLASS(6)= 1
NO. OF CRYSTALS IN CLASS(6)= 0
MASS UNDER-SIZE IN CLASS(7)= 1
NO. OF CRYSTALS IN CLASS(7)= 0
MASS UNDER-SIZE IN CLASS(8)= 1
NO. OF CRYSTALS IN CLASS(8)= 0
MASS UNDER-SIZE IN CLASS(9)= 1
NO. OF CRYSTALS IN CLASS(9)= 0
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 0
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=88.4583333333333min
Total number of crystals=16399
MASS UNDER-SIZE IN CLASS(0)= 4.921402E-02
NO. OF CRYSTALS IN CLASS(0)= 14085
MASS UNDER-SIZE IN CLASS(1)= .2685087
NO. OF CRYSTALS IN CLASS(1)= 1743
MASS UNDER-SIZE IN CLASS(2)= .3725953
NO. OF CRYSTALS IN CLASS(2)= 204
MASS UNDER-SIZE IN CLASS(3)= .4912164
NO. OF CRYSTALS IN CLASS(3)= 138

MASS UNDER-SIZE IN CLASS(4)= .6596665
NO. OF CRYSTALS IN CLASS(4)= 116
MASS UNDER-SIZE IN CLASS(5)= .8204942
NO. OF CRYSTALS IN CLASS(5)= 67
MASS UNDER-SIZE IN CLASS(6)= 1
NO. OF CRYSTALS IN CLASS(6)= 46
MASS UNDER-SIZE IN CLASS(7)= 1
NO. OF CRYSTALS IN CLASS(7)= 0
MASS UNDER-SIZE IN CLASS(8)= 1
NO. OF CRYSTALS IN CLASS(8)= 0
MASS UNDER-SIZE IN CLASS(9)= 1
NO. OF CRYSTALS IN CLASS(9)= 0
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 0
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=104.541666666667min
Total number of crystals=16539
MASS UNDER-SIZE IN CLASS(0)= 3.586309E-02
NO. OF CRYSTALS IN CLASS(0)= 14117
MASS UNDER-SIZE IN CLASS(1)= .1983787
NO. OF CRYSTALS IN CLASS(1)= 1781
MASS UNDER-SIZE IN CLASS(2)= .2904016
NO. OF CRYSTALS IN CLASS(2)= 248
MASS UNDER-SIZE IN CLASS(3)= .3660758
NO. OF CRYSTALS IN CLASS(3)= 118
MASS UNDER-SIZE IN CLASS(4)= .4767982
NO. OF CRYSTALS IN CLASS(4)= 106
MASS UNDER-SIZE IN CLASS(5)= .6154193
NO. OF CRYSTALS IN CLASS(5)= 78
MASS UNDER-SIZE IN CLASS(6)= .7830982
NO. OF CRYSTALS IN CLASS(6)= 55
MASS UNDER-SIZE IN CLASS(7)= .9038921
NO. OF CRYSTALS IN CLASS(7)= 24
MASS UNDER-SIZE IN CLASS(8)= 1
NO. OF CRYSTALS IN CLASS(8)= 12
MASS UNDER-SIZE IN CLASS(9)= 1
NO. OF CRYSTALS IN CLASS(9)= 0
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 0
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1

NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=120.625min
Total number of crystals=16593
MASS UNDER-SIZE IN CLASS(0)= 3.257208E-02
NO. OF CRYSTALS IN CLASS(0)= 14211
MASS UNDER-SIZE IN CLASS(1)= .1745434
NO. OF CRYSTALS IN CLASS(1)= 1737
MASS UNDER-SIZE IN CLASS(2)= .264504
NO. OF CRYSTALS IN CLASS(2)= 268
MASS UNDER-SIZE IN CLASS(3)= .3311314
NO. OF CRYSTALS IN CLASS(3)= 116
MASS UNDER-SIZE IN CLASS(4)= .4331805
NO. OF CRYSTALS IN CLASS(4)= 105
MASS UNDER-SIZE IN CLASS(5)= .520781
NO. OF CRYSTALS IN CLASS(5)= 53
MASS UNDER-SIZE IN CLASS(6)= .6587906
NO. OF CRYSTALS IN CLASS(6)= 50
MASS UNDER-SIZE IN CLASS(7)= .8205546
NO. OF CRYSTALS IN CLASS(7)= 34
MASS UNDER-SIZE IN CLASS(8)= .9154645
NO. OF CRYSTALS IN CLASS(8)= 12
MASS UNDER-SIZE IN CLASS(9)= 1
NO. OF CRYSTALS IN CLASS(9)= 7
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 0
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=136.7083333333333min
Total number of crystals=16748
MASS UNDER-SIZE IN CLASS(0)= 2.621139E-02
NO. OF CRYSTALS IN CLASS(0)= 14288
MASS UNDER-SIZE IN CLASS(1)= .1489457
NO. OF CRYSTALS IN CLASS(1)= 1811
MASS UNDER-SIZE IN CLASS(2)= .203663
NO. OF CRYSTALS IN CLASS(2)= 206
MASS UNDER-SIZE IN CLASS(3)= .2648363
NO. OF CRYSTALS IN CLASS(3)= 134
MASS UNDER-SIZE IN CLASS(4)= .3702528

NO. OF CRYSTALS IN CLASS(4)= 135
MASS UNDER-SIZE IN CLASS(5)= .4542509
NO. OF CRYSTALS IN CLASS(5)= 65
MASS UNDER-SIZE IN CLASS(6)= .561769
NO. OF CRYSTALS IN CLASS(6)= 48
MASS UNDER-SIZE IN CLASS(7)= .6519904
NO. OF CRYSTALS IN CLASS(7)= 24
MASS UNDER-SIZE IN CLASS(8)= .757058
NO. OF CRYSTALS IN CLASS(8)= 17
MASS UNDER-SIZE IN CLASS(9)= .8973248
NO. OF CRYSTALS IN CLASS(9)= 14
MASS UNDER-SIZE IN CLASS(10)= 1
NO. OF CRYSTALS IN CLASS(10)= 6
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=152.791666666667min
Total number of crystals=16610
MASS UNDER-SIZE IN CLASS(0)= 2.465028E-02
NO. OF CRYSTALS IN CLASS(0)= 14151
MASS UNDER-SIZE IN CLASS(1)= .1387122
NO. OF CRYSTALS IN CLASS(1)= 1796
MASS UNDER-SIZE IN CLASS(2)= .1957899
NO. OF CRYSTALS IN CLASS(2)= 221
MASS UNDER-SIZE IN CLASS(3)= .2577045
NO. OF CRYSTALS IN CLASS(3)= 144
MASS UNDER-SIZE IN CLASS(4)= .3457942
NO. OF CRYSTALS IN CLASS(4)= 119
MASS UNDER-SIZE IN CLASS(5)= .4186654
NO. OF CRYSTALS IN CLASS(5)= 59
MASS UNDER-SIZE IN CLASS(6)= .5372391
NO. OF CRYSTALS IN CLASS(6)= 57
MASS UNDER-SIZE IN CLASS(7)= .6517985
NO. OF CRYSTALS IN CLASS(7)= 33
MASS UNDER-SIZE IN CLASS(8)= .735014
NO. OF CRYSTALS IN CLASS(8)= 14
MASS UNDER-SIZE IN CLASS(9)= .7967467
NO. OF CRYSTALS IN CLASS(9)= 7
MASS UNDER-SIZE IN CLASS(10)= .882637
NO. OF CRYSTALS IN CLASS(10)= 5
MASS UNDER-SIZE IN CLASS(11)= 1
NO. OF CRYSTALS IN CLASS(11)= 4
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 0
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0

MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=160.833333333333min
Total number of crystals=16546
MASS UNDER-SIZE IN CLASS(0)= 2.485323E-02
NO. OF CRYSTALS IN CLASS(0)= 14098
MASS UNDER-SIZE IN CLASS(1)= .136915
NO. OF CRYSTALS IN CLASS(1)= 1775
MASS UNDER-SIZE IN CLASS(2)= .1982689
NO. OF CRYSTALS IN CLASS(2)= 234
MASS UNDER-SIZE IN CLASS(3)= .2586189
NO. OF CRYSTALS IN CLASS(3)= 136
MASS UNDER-SIZE IN CLASS(4)= .3448039
NO. OF CRYSTALS IN CLASS(4)= 116
MASS UNDER-SIZE IN CLASS(5)= .4366216
NO. OF CRYSTALS IN CLASS(5)= 73
MASS UNDER-SIZE IN CLASS(6)= .5448447
NO. OF CRYSTALS IN CLASS(6)= 50
MASS UNDER-SIZE IN CLASS(7)= .6825542
NO. OF CRYSTALS IN CLASS(7)= 36
MASS UNDER-SIZE IN CLASS(8)= .7460194
NO. OF CRYSTALS IN CLASS(8)= 11
MASS UNDER-SIZE IN CLASS(9)= .853159
NO. OF CRYSTALS IN CLASS(9)= 11
MASS UNDER-SIZE IN CLASS(10)= .8796564
NO. OF CRYSTALS IN CLASS(10)= 2
MASS UNDER-SIZE IN CLASS(11)= .960243
NO. OF CRYSTALS IN CLASS(11)= 3
MASS UNDER-SIZE IN CLASS(12)= .9999999
NO. OF CRYSTALS IN CLASS(12)= 1
MASS UNDER-SIZE IN CLASS(13)= .9999999
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= .9999999
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= .9999999
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= .9999999
NO. OF CRYSTALS IN CLASS(16)= 0
----------steady state----------

time=168.875min
Total number of crystals=16595
MASS UNDER-SIZE IN CLASS(0)= 2.484041E-02
NO. OF CRYSTALS IN CLASS(0)= 14205
MASS UNDER-SIZE IN CLASS(1)= .1341636
NO. OF CRYSTALS IN CLASS(1)= 1752
MASS UNDER-SIZE IN CLASS(2)= .1860862
NO. OF CRYSTALS IN CLASS(2)= 207
MASS UNDER-SIZE IN CLASS(3)= .2406228
NO. OF CRYSTALS IN CLASS(3)= 128
MASS UNDER-SIZE IN CLASS(4)= .3329415

NO. OF CRYSTALS IN CLASS(4)= 130
MASS UNDER-SIZE IN CLASS(5)= .4034149
NO. OF CRYSTALS IN CLASS(5)= 59
MASS UNDER-SIZE IN CLASS(6)= .5218537
NO. OF CRYSTALS IN CLASS(6)= 57
MASS UNDER-SIZE IN CLASS(7)= .5977187
NO. OF CRYSTALS IN CLASS(7)= 21
MASS UNDER-SIZE IN CLASS(8)= .7141598
NO. OF CRYSTALS IN CLASS(8)= 19
MASS UNDER-SIZE IN CLASS(9)= .8020496
NO. OF CRYSTALS IN CLASS(9)= 9
MASS UNDER-SIZE IN CLASS(10)= .8831388
NO. OF CRYSTALS IN CLASS(10)= 5
MASS UNDER-SIZE IN CLASS(11)= .9477104
NO. OF CRYSTALS IN CLASS(11)= 2
MASS UNDER-SIZE IN CLASS(12)= 1
NO. OF CRYSTALS IN CLASS(12)= 1
MASS UNDER-SIZE IN CLASS(13)= 1
NO. OF CRYSTALS IN CLASS(13)= 0
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

time=176.916666666667min
Total number of crystals=16532
MASS UNDER-SIZE IN CLASS(0)= 2.267442E-02
NO. OF CRYSTALS IN CLASS(0)= 14121
MASS UNDER-SIZE IN CLASS(1)= .1263279
NO. OF CRYSTALS IN CLASS(1)= 1772
MASS UNDER-SIZE IN CLASS(2)= .1795191
NO. OF CRYSTALS IN CLASS(2)= 226
MASS UNDER-SIZE IN CLASS(3)= .2279695
NO. OF CRYSTALS IN CLASS(3)= 123
MASS UNDER-SIZE IN CLASS(4)= .3045576
NO. OF CRYSTALS IN CLASS(4)= 115
MASS UNDER-SIZE IN CLASS(5)= .3742453
NO. OF CRYSTALS IN CLASS(5)= 62
MASS UNDER-SIZE IN CLASS(6)= .4628326
NO. OF CRYSTALS IN CLASS(6)= 48
MASS UNDER-SIZE IN CLASS(7)= .5641868
NO. OF CRYSTALS IN CLASS(7)= 30
MASS UNDER-SIZE IN CLASS(8)= .6210552
NO. OF CRYSTALS IN CLASS(8)= 10
MASS UNDER-SIZE IN CLASS(9)= .7789322
NO. OF CRYSTALS IN CLASS(9)= 17
MASS UNDER-SIZE IN CLASS(10)= .8452032
NO. OF CRYSTALS IN CLASS(10)= 4
MASS UNDER-SIZE IN CLASS(11)= .8929779
NO. OF CRYSTALS IN CLASS(11)= 2
MASS UNDER-SIZE IN CLASS(12)= .9338817
NO. OF CRYSTALS IN CLASS(12)= 1
MASS UNDER-SIZE IN CLASS(13)= .9999999
NO. OF CRYSTALS IN CLASS(13)= 1

MASS UNDER-SIZE IN CLASS(14)= .9999999
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= .9999999
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= .9999999
NO. OF CRYSTALS IN CLASS(16)= 0

time=193min
Total number of crystals=16730
MASS UNDER-SIZE IN CLASS(0)= .022207
NO. OF CRYSTALS IN CLASS(0)= 14320
MASS UNDER-SIZE IN CLASS(1)= .1178303
NO. OF CRYSTALS IN CLASS(1)= 1758
MASS UNDER-SIZE IN CLASS(2)= .1651853
NO. OF CRYSTALS IN CLASS(2)= 215
MASS UNDER-SIZE IN CLASS(3)= .221174
NO. OF CRYSTALS IN CLASS(3)= 148
MASS UNDER-SIZE IN CLASS(4)= .2998221
NO. OF CRYSTALS IN CLASS(4)= 125
MASS UNDER-SIZE IN CLASS(5)= .3530285
NO. OF CRYSTALS IN CLASS(5)= 50
MASS UNDER-SIZE IN CLASS(6)= .4655669
NO. OF CRYSTALS IN CLASS(6)= 60
MASS UNDER-SIZE IN CLASS(7)= .5472362
NO. OF CRYSTALS IN CLASS(7)= 24
MASS UNDER-SIZE IN CLASS(8)= .5923142
NO. OF CRYSTALS IN CLASS(8)= 9
MASS UNDER-SIZE IN CLASS(9)= .6790065
NO. OF CRYSTALS IN CLASS(9)= 11
MASS UNDER-SIZE IN CLASS(10)= .7668558
NO. OF CRYSTALS IN CLASS(10)= 6
MASS UNDER-SIZE IN CLASS(11)= .7890013
NO. OF CRYSTALS IN CLASS(11)= 1
MASS UNDER-SIZE IN CLASS(12)= .8221673
NO. OF CRYSTALS IN CLASS(12)= 1
MASS UNDER-SIZE IN CLASS(13)= .8917708
NO. OF CRYSTALS IN CLASS(13)= 1
MASS UNDER-SIZE IN CLASS(14)= 1
NO. OF CRYSTALS IN CLASS(14)= 1
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 0
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

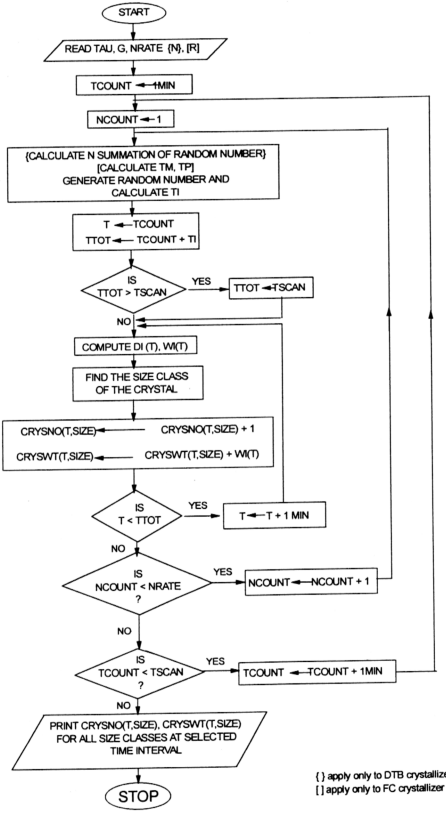
time=209.083333333333min
Total number of crystals=16647
MASS UNDER-SIZE IN CLASS(0)= 2.091017E-02
NO. OF CRYSTALS IN CLASS(0)= 14206
MASS UNDER-SIZE IN CLASS(1)= .1127919
NO. OF CRYSTALS IN CLASS(1)= 1797
MASS UNDER-SIZE IN CLASS(2)= .1585201
NO. OF CRYSTALS IN CLASS(2)= 217
MASS UNDER-SIZE IN CLASS(3)= .2052197
NO. OF CRYSTALS IN CLASS(3)= 131
MASS UNDER-SIZE IN CLASS(4)= .2774667
NO. OF CRYSTALS IN CLASS(4)= 120

MASS UNDER-SIZE IN CLASS(5)= .3401705
NO. OF CRYSTALS IN CLASS(5)= 62
MASS UNDER-SIZE IN CLASS(6)= .4493751
NO. OF CRYSTALS IN CLASS(6)= 65
MASS UNDER-SIZE IN CLASS(7)= .5003447
NO. OF CRYSTALS IN CLASS(7)= 16
MASS UNDER-SIZE IN CLASS(8)= .5619428
NO. OF CRYSTALS IN CLASS(8)= 13
MASS UNDER-SIZE IN CLASS(9)= .639773
NO. OF CRYSTALS IN CLASS(9)= 10
MASS UNDER-SIZE IN CLASS(10)= .7343737
NO. OF CRYSTALS IN CLASS(10)= 7
MASS UNDER-SIZE IN CLASS(11)= .7343737
NO. OF CRYSTALS IN CLASS(11)= 0
MASS UNDER-SIZE IN CLASS(12)= .7737661
NO. OF CRYSTALS IN CLASS(12)= 1
MASS UNDER-SIZE IN CLASS(13)= .8307002
NO. OF CRYSTALS IN CLASS(13)= 1
MASS UNDER-SIZE IN CLASS(14)= .8307002
NO. OF CRYSTALS IN CLASS(14)= 0
MASS UNDER-SIZE IN CLASS(15)= 1
NO. OF CRYSTALS IN CLASS(15)= 1
MASS UNDER-SIZE IN CLASS(16)= 1
NO. OF CRYSTALS IN CLASS(16)= 0

APPENDIX III

Appendix III.1

Outline of Simulation Algorithm for Transient CSD in DTB and FC Crystallizers



Appendix III.2

Sample Program and Results for Transient CSD in DTB Crystallizer

Declaration

```
Dim wtfr(0 To 5) As Single
Dim cmwtfr(0 To 5), Ben(0 To 5) As Single
Dim wet(0 To 5) As Single
Dim cr(1 To 1800, 0 To 5) As Integer
Dim wt(1 To 1800, 0 To 5)  As Single
Dim tau, G, Tscan, t1, t2, nt, Num, size As Single
Dim top, bottom, count, A, rms As Single
Dim cryswt As Single
```

Main Program

```
Sub main ()

nstage = Val(stage.Text1.Text)
tau = Val(stage.Text2.Text) * 60
G = Val(stage.Text3.Text) / 60
Num = Val(stage.Text4.Text)

Open "sim_dtb" For Output As #1
Print #1, "Results of transient MC Simulation"
Print #1, "Number of stage =" & nstage
Print #1, "Residence time in an imperfectly mixed crystallizer =" & tau / 60;
" hr"
Print #1, "Average growth rate of a crystal = " & G * 60; " mm/hr"
Print #1, "Compared with Run No. " & Num; " Bennett and van Buren"

nt = 60
Print #1, "Nucleation rate =" & nt; " nuclei/min"

Ben(0) = 100
Ben(1) = 99
Ben(2) = 96
Ben(3) = 83
Ben(4) = 54
Ben(5) = 17
```

```
Tscan = 13 * tau
For t1 = 1 To Tscan
  For n = 1 To nt
    If nstage = 1 Then
      t2 = -tau * Log(Rnd)
    Else
    For st = 1 To nstage
      sumrnd = sumrnd + Log(Rnd)
    Next
      t2 = -(tau / nstage) * sumrnd
    End If
    ttot = t1 + t2
    If ttot > Tscan Then ttot = Tscan
    For times = t1 To ttot Step 1
      size = G * (times - t1)
      If size > .208 Then
        GoTo 260
      Else
        s = 0
        GoTo 310
      End If
260   If size > .295 Then
        GoTo 270
      Else
        s = 1
        GoTo 310
      End If
270   If size > .417 Then
        GoTo 280
      Else
        s = 2
      GoTo 310
      End If
280   If size > .589 Then
        GoTo 290
      Else
        s = 3
        GoTo 310
      End If
290   If size < .833 Then
        s = 4
      Else
        s = 5
      End If
310   cr(times, s) = cr(times, s) + 1
      wt(times, s) = wt(times, s) + (size ^ 3)
```

```
      Next
      n5 = n5 + 1
      Debug.Print n5
      sumrnd = 0
    Next
  Next
  For times = 1 To Tscan Step 30
    For j = 0 To 5
      wet(j) = wt(times, j)
      cryswt = cryswt + wet(j)
    Next
    If cryswt = 0 Then
      GoTo 4
    Else
      wtfr(5) = (wet(5) / cryswt) * 100
      cmwtfr(5) = wtfr(5)
      For j = 4 To 0 Step -1
        wtfr(j) = (wet(j) / cryswt) * 100
        cmwtfr(j) = cmwtfr(j + 1) + wtfr(j)
      Next
      For j = 0 To 5
        top = top + (cmwtfr(j) - Ben(j)) ^ 2
        bottom = bottom + (cmwtfr(j) - Ben(j))
        count = count + 1
      Next
      A = top / bottom ^ 2
      rms = (top / count) ^ (1 / 2)
      Print #1, "time =" & times / 60; " hour"
      Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 0)
      Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 1)
      Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 2)
      Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 3)
      Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 4)
      Print #1, " No of crystals in tyler mesh 14 = " & cr(times, 5)
      Print #1, " cmwtfr in tyler mesh 65 = " & cmwtfr(0)
      Print #1, " cmwtfr in tyler mesh 48 = " & cmwtfr(1)
      Print #1, " cmwtfr in tyler mesh 35 = " & cmwtfr(2)
      Print #1, " cmwtfr in tyler mesh 28 = " & cmwtfr(3)
      Print #1, " cmwtfr in tyler mesh 20 = " & cmwtfr(4)
      Print #1, " cmwtfr in tyler mesh 14 = " & cmwtfr(5)
      Print #1, "Sandler A-test = " & A
      Print #1, "rms = " & rms
      Print #1,
      cryswt = 0
    End If
4  top = 0: bottom = 0: count = 0
```

```
Next

End Sub
```

Sample of results (for run no. 113 with N=1)

```
Results of transient MC Simulation
Number of stage =1
Residence time in an imperfectly mixed crystallizer =5.9 hr
Average growth rate of a crystal = .0485 mm/hr
Compared with Run No. 113 Bennett and van Buren
Nucleation rate =60 nuclei/min

time =1.01666666666667 hour
 No of crystals in tyler mesh 65 = 3364
 No of crystals in tyler mesh 48 = 0
 No of crystals in tyler mesh 35 = 0
 No of crystals in tyler mesh 28 = 0
 No of crystals in tyler mesh 20 = 0
 No of crystals in tyler mesh 14 = 0
 cmwtfr in tyler mesh 65 = 100
 cmwtfr in tyler mesh 48 = 0
 cmwtfr in tyler mesh 35 = 0
 cmwtfr in tyler mesh 28 = 0
 cmwtfr in tyler mesh 20 = 0
 cmwtfr in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time =2.01666666666667 hour
 No of crystals in tyler mesh 65 = 6066
 No of crystals in tyler mesh 48 = 0
 No of crystals in tyler mesh 35 = 0
 No of crystals in tyler mesh 28 = 0
 No of crystals in tyler mesh 20 = 0
 No of crystals in tyler mesh 14 = 0
 cmwtfr in tyler mesh 65 = 100
 cmwtfr in tyler mesh 48 = 0
 cmwtfr in tyler mesh 35 = 0
 cmwtfr in tyler mesh 28 = 0
 cmwtfr in tyler mesh 20 = 0
 cmwtfr in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time =3.01666666666667 hour
 No of crystals in tyler mesh 65 = 8395
 No of crystals in tyler mesh 48 = 0
 No of crystals in tyler mesh 35 = 0
 No of crystals in tyler mesh 28 = 0
 No of crystals in tyler mesh 20 = 0
 No of crystals in tyler mesh 14 = 0
 cmwtfr in tyler mesh 65 = 100
```

cmwtfr in tyler mesh 48 = 0
cmwtfr in tyler mesh 35 = 0
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time =4.01666666666667 hour
No of crystals in tyler mesh 65 = 10322
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 0
cmwtfr in tyler mesh 35 = 0
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time =5.01666666666667 hour
No of crystals in tyler mesh 65 = 10763
No of crystals in tyler mesh 48 = 1133
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 40.37654
cmwtfr in tyler mesh 35 = 0
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .217451498342597
rms = 62.65625

time =6.01666666666667 hour
No of crystals in tyler mesh 65 = 10788
No of crystals in tyler mesh 48 = 2528
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 67.1923
cmwtfr in tyler mesh 35 = 0
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .231498895731626
rms = 59.38107

time =7.01666666666667 hour
No of crystals in tyler mesh 65 = 10848
No of crystals in tyler mesh 48 = 2655
No of crystals in tyler mesh 35 = 1056
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 79.66666
cmwtfr in tyler mesh 35 = 35.23663
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .235331047807496
rms = 50.42162

time =8.01666666666667 hour
No of crystals in tyler mesh 65 = 10769
No of crystals in tyler mesh 48 = 2683
No of crystals in tyler mesh 35 = 2084
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 86.82887
cmwtfr in tyler mesh 35 = 57.25718
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .255818766153271
rms = 46.54482

time =9.01666666666667 hour
No of crystals in tyler mesh 65 = 10932
No of crystals in tyler mesh 48 = 2561
No of crystals in tyler mesh 35 = 2654
No of crystals in tyler mesh 28 = 331
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 90.4527
cmwtfr in tyler mesh 35 = 70.35413
cmwtfr in tyler mesh 28 = 12.62663
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .267987329418565
rms = 41.43668

time =10.0166666666667 hour
No of crystals in tyler mesh 65 = 10928
No of crystals in tyler mesh 48 = 2601
No of crystals in tyler mesh 35 = 2645
No of crystals in tyler mesh 28 = 1076
No of crystals in tyler mesh 20 = 0

No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 92.96579
cmwtfr in tyler mesh 35 = 78.1534
cmwtfr in tyler mesh 28 = 35.44728
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .278980942963248
rms = 35.13356

time =11.0166666666667 hour
No of crystals in tyler mesh 65 = 10816
No of crystals in tyler mesh 48 = 2697
No of crystals in tyler mesh 35 = 2540
No of crystals in tyler mesh 28 = 1683
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 94.61479
cmwtfr in tyler mesh 35 = 82.45538
cmwtfr in tyler mesh 28 = 50.3029
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .305338334079631
rms = 32.06206

time =12.0166666666667 hour
No of crystals in tyler mesh 65 = 10839
No of crystals in tyler mesh 48 = 2605
No of crystals in tyler mesh 35 = 2593
No of crystals in tyler mesh 28 = 2174
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 95.61815
cmwtfr in tyler mesh 35 = 86.2231
cmwtfr in tyler mesh 28 = 60.15929
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .344574898059108
rms = 30.55446

time =13.0166666666667 hour
No of crystals in tyler mesh 65 = 10699
No of crystals in tyler mesh 48 = 2586
No of crystals in tyler mesh 35 = 2630
No of crystals in tyler mesh 28 = 2224
No of crystals in tyler mesh 20 = 349
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 96.38834
cmwtfr in tyler mesh 35 = 88.7432
cmwtfr in tyler mesh 28 = 66.92623
cmwtfr in tyler mesh 20 = 14.66681
cmwtfr in tyler mesh 14 = 0

Sandler A-test = .35400225351712
rms = 24.96414

time =14.0166666666667 hour
No of crystals in tyler mesh 65 = 10740
No of crystals in tyler mesh 48 = 2638
No of crystals in tyler mesh 35 = 2633
No of crystals in tyler mesh 28 = 2162
No of crystals in tyler mesh 20 = 731
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 97.00966
cmwtfr in tyler mesh 35 = 90.35435
cmwtfr in tyler mesh 28 = 71.62055
cmwtfr in tyler mesh 20 = 29.12073
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .371725209350622
rms = 20.25963

time =15.0166666666667 hour
No of crystals in tyler mesh 65 = 10787
No of crystals in tyler mesh 48 = 2612
No of crystals in tyler mesh 35 = 2606
No of crystals in tyler mesh 28 = 2223
No of crystals in tyler mesh 20 = 1012
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 97.45329
cmwtfr in tyler mesh 35 = 91.82536
cmwtfr in tyler mesh 28 = 75.98595
cmwtfr in tyler mesh 20 = 38.55569
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .432924968501883
rms = 17.64258

time =16.0166666666667 hour
No of crystals in tyler mesh 65 = 10846
No of crystals in tyler mesh 48 = 2561
No of crystals in tyler mesh 35 = 2645
No of crystals in tyler mesh 28 = 2199
No of crystals in tyler mesh 20 = 1293
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 97.75908
cmwtfr in tyler mesh 35 = 92.94943
cmwtfr in tyler mesh 28 = 78.99797
cmwtfr in tyler mesh 20 = 46.8991
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .545798245116729
rms = 15.95329

time =17.0166666666667 hour
No of crystals in tyler mesh 65 = 10872
No of crystals in tyler mesh 48 = 2554
No of crystals in tyler mesh 35 = 2587

cmwtfr in tyler mesh 20 = 64.92086
cmwtfr in tyler mesh 14 = 23.34954
Sandler A-test = 173.401057506013
rms = 5.932586

time =21.0166666666667 hour
No of crystals in tyler mesh 65 = 10963
No of crystals in tyler mesh 48 = 2625
No of crystals in tyler mesh 35 = 2621
No of crystals in tyler mesh 28 = 2130
No of crystals in tyler mesh 20 = 1587
No of crystals in tyler mesh 14 = 540
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 98.60596
cmwtfr in tyler mesh 35 = 95.61789
cmwtfr in tyler mesh 28 = 87.32194
cmwtfr in tyler mesh 20 = 68.40643
cmwtfr in tyler mesh 14 = 29.76304
Sandler A-test = 1.30533295248916
rms = 4.764687
----------Closest to experimental result----------
time =21.5166666666667 hour
No of crystals in tyler mesh 65 = 10996
No of crystals in tyler mesh 48 = 2604
No of crystals in tyler mesh 35 = 2619
No of crystals in tyler mesh 28 = 2146
No of crystals in tyler mesh 20 = 1577
No of crystals in tyler mesh 14 = 587
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 98.67246
cmwtfr in tyler mesh 35 = 95.85206
cmwtfr in tyler mesh 28 = 87.92413
cmwtfr in tyler mesh 20 = 69.55965
cmwtfr in tyler mesh 14 = 32.1621
Sandler A-test = .642417705094717
rms = 4.800373

time =22.0166666666667 hour
No of crystals in tyler mesh 65 = 10921
No of crystals in tyler mesh 48 = 2637
No of crystals in tyler mesh 35 = 2590
No of crystals in tyler mesh 28 = 2120
No of crystals in tyler mesh 20 = 1563
No of crystals in tyler mesh 14 = 639
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 98.71183
cmwtfr in tyler mesh 35 = 95.92284
cmwtfr in tyler mesh 28 = 88.24663
cmwtfr in tyler mesh 20 = 70.72864
cmwtfr in tyler mesh 14 = 34.91081
Sandler A-test = .412247185801762
rms = 4.985752

time =23.0166666666667 hour
No of crystals in tyler mesh 65 = 10993

No of crystals in tyler mesh 48 = 2618
No of crystals in tyler mesh 35 = 2584
No of crystals in tyler mesh 28 = 2102
No of crystals in tyler mesh 20 = 1565
No of crystals in tyler mesh 14 = 715
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 98.77679
cmwtfr in tyler mesh 35 = 96.16421
cmwtfr in tyler mesh 28 = 88.92896
cmwtfr in tyler mesh 20 = 72.74474
cmwtfr in tyler mesh 14 = 38.91863
Sandler A-test = .307669609109685
rms = 5.895172

time =24.0166666666667 hour
No of crystals in tyler mesh 65 = 10900
No of crystals in tyler mesh 48 = 2636
No of crystals in tyler mesh 35 = 2595
No of crystals in tyler mesh 28 = 2161
No of crystals in tyler mesh 20 = 1515
No of crystals in tyler mesh 14 = 762
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 98.82693
cmwtfr in tyler mesh 35 = 96.31202
cmwtfr in tyler mesh 28 = 89.35643
cmwtfr in tyler mesh 20 = 73.49812
cmwtfr in tyler mesh 14 = 41.94675
Sandler A-test = .281289620196995
rms = 6.590969

time =25.0166666666667 hour
No of crystals in tyler mesh 65 = 10932
No of crystals in tyler mesh 48 = 2662
No of crystals in tyler mesh 35 = 2567
No of crystals in tyler mesh 28 = 2213
No of crystals in tyler mesh 20 = 1474
No of crystals in tyler mesh 14 = 819
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 98.87888
cmwtfr in tyler mesh 35 = 96.46234
cmwtfr in tyler mesh 28 = 89.91338
cmwtfr in tyler mesh 20 = 74.24378
cmwtfr in tyler mesh 14 = 44.98783
Sandler A-test = .273804866718859
rms = 7.473805

time =26.0166666666667 hour
No of crystals in tyler mesh 65 = 10930
No of crystals in tyler mesh 48 = 2635
No of crystals in tyler mesh 35 = 2577
No of crystals in tyler mesh 28 = 2310
No of crystals in tyler mesh 20 = 1455
No of crystals in tyler mesh 14 = 870
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 98.93386

cmwtfr in tyler mesh 35 = 96.64212
cmwtfr in tyler mesh 28 = 90.38899
cmwtfr in tyler mesh 20 = 74.55151
cmwtfr in tyler mesh 14 = 47.19962
Sandler A-test = .272589833023175
rms = 8.14565

time =27.0166666666667 hour
No of crystals in tyler mesh 65 = 10934
No of crystals in tyler mesh 48 = 2654
No of crystals in tyler mesh 35 = 2577
No of crystals in tyler mesh 28 = 2245
No of crystals in tyler mesh 20 = 1554
No of crystals in tyler mesh 14 = 915
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 98.99896
cmwtfr in tyler mesh 35 = 96.83035
cmwtfr in tyler mesh 28 = 90.94272
cmwtfr in tyler mesh 20 = 76.51506
cmwtfr in tyler mesh 14 = 49.32563
Sandler A-test = .277779432968805
rms = 9.276408

time =28.0166666666667 hour
No of crystals in tyler mesh 65 = 10972
No of crystals in tyler mesh 48 = 2684
No of crystals in tyler mesh 35 = 2555
No of crystals in tyler mesh 28 = 2226
No of crystals in tyler mesh 20 = 1571
No of crystals in tyler mesh 14 = 968
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.03942
cmwtfr in tyler mesh 35 = 96.94078
cmwtfr in tyler mesh 28 = 91.36646
cmwtfr in tyler mesh 20 = 77.73923
cmwtfr in tyler mesh 14 = 51.80339
Sandler A-test = .283007368999352
rms = 10.29209

time =29.0166666666667 hour
No of crystals in tyler mesh 65 = 10975
No of crystals in tyler mesh 48 = 2669
No of crystals in tyler mesh 35 = 2645
No of crystals in tyler mesh 28 = 2223
No of crystals in tyler mesh 20 = 1598
No of crystals in tyler mesh 14 = 999
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.08818
cmwtfr in tyler mesh 35 = 97.11533
cmwtfr in tyler mesh 28 = 91.66582
cmwtfr in tyler mesh 20 = 78.76692
cmwtfr in tyler mesh 14 = 53.58736
Sandler A-test = .286358662672757
rms = 11.08127

time =30.0166666666667 hour
No of crystals in tyler mesh 65 = 10980
No of crystals in tyler mesh 48 = 2650
No of crystals in tyler mesh 35 = 2659
No of crystals in tyler mesh 28 = 2199
No of crystals in tyler mesh 20 = 1578
No of crystals in tyler mesh 14 = 1020
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.12492
cmwtfr in tyler mesh 35 = 97.23554
cmwtfr in tyler mesh 28 = 91.95921
cmwtfr in tyler mesh 20 = 79.73531
cmwtfr in tyler mesh 14 = 55.50277
Sandler A-test = .290540418941053
rms = 11.89558

time =31.0166666666667 hour
No of crystals in tyler mesh 65 = 10974
No of crystals in tyler mesh 48 = 2610
No of crystals in tyler mesh 35 = 2622
No of crystals in tyler mesh 28 = 2189
No of crystals in tyler mesh 20 = 1594
No of crystals in tyler mesh 14 = 1019
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.1285
cmwtfr in tyler mesh 35 = 97.29533
cmwtfr in tyler mesh 28 = 92.15073
cmwtfr in tyler mesh 20 = 80.37537
cmwtfr in tyler mesh 14 = 56.15508
Sandler A-test = .292055495276171
rms = 12.26792

time =32.0166666666667 hour
No of crystals in tyler mesh 65 = 10865
No of crystals in tyler mesh 48 = 2634
No of crystals in tyler mesh 35 = 2664
No of crystals in tyler mesh 28 = 2236
No of crystals in tyler mesh 20 = 1531
No of crystals in tyler mesh 14 = 1053
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.16982
cmwtfr in tyler mesh 35 = 97.39096
cmwtfr in tyler mesh 28 = 92.32141
cmwtfr in tyler mesh 20 = 80.48339
cmwtfr in tyler mesh 14 = 57.89549
Sandler A-test = .295282737578631
rms = 12.81382

time =33.0166666666667 hour
No of crystals in tyler mesh 65 = 10855
No of crystals in tyler mesh 48 = 2718
No of crystals in tyler mesh 35 = 2626
No of crystals in tyler mesh 28 = 2237
No of crystals in tyler mesh 20 = 1546
No of crystals in tyler mesh 14 = 1049

cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.1958
cmwtfr in tyler mesh 35 = 97.36531
cmwtfr in tyler mesh 28 = 92.40554
cmwtfr in tyler mesh 20 = 80.66828
cmwtfr in tyler mesh 14 = 58.18551
Sandler A-test = .295947630431022
rms = 12.95247

time =34.0166666666667 hour
No of crystals in tyler mesh 65 = 10876
No of crystals in tyler mesh 48 = 2636
No of crystals in tyler mesh 35 = 2635
No of crystals in tyler mesh 28 = 2248
No of crystals in tyler mesh 20 = 1517
No of crystals in tyler mesh 14 = 1105
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.21347
cmwtfr in tyler mesh 35 = 97.49135
cmwtfr in tyler mesh 28 = 92.68999
cmwtfr in tyler mesh 20 = 81.35741
cmwtfr in tyler mesh 14 = 60.17699
Sandler A-test = .300315472150746
rms = 13.74321

time =35.0166666666667 hour
No of crystals in tyler mesh 65 = 10938
No of crystals in tyler mesh 48 = 2561
No of crystals in tyler mesh 35 = 2661
No of crystals in tyler mesh 28 = 2254
No of crystals in tyler mesh 20 = 1503
No of crystals in tyler mesh 14 = 1103
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.20908
cmwtfr in tyler mesh 35 = 97.57296
cmwtfr in tyler mesh 28 = 92.81255
cmwtfr in tyler mesh 20 = 81.55589
cmwtfr in tyler mesh 14 = 60.82286
Sandler A-test = .301426798210528
rms = 14.00264

time =35.5166666666667 hour
No of crystals in tyler mesh 65 = 10869
No of crystals in tyler mesh 48 = 2644
No of crystals in tyler mesh 35 = 2628
No of crystals in tyler mesh 28 = 2241
No of crystals in tyler mesh 20 = 1514
No of crystals in tyler mesh 14 = 1115
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.23483
cmwtfr in tyler mesh 35 = 97.58866
cmwtfr in tyler mesh 28 = 92.91881
cmwtfr in tyler mesh 20 = 81.80238
cmwtfr in tyler mesh 14 = 61.15747
Sandler A-test = .301837614028042

rms = 14.17565

time =36.0166666666667 hour
No of crystals in tyler mesh 65 = 10923
No of crystals in tyler mesh 48 = 2653
No of crystals in tyler mesh 35 = 2581
No of crystals in tyler mesh 28 = 2275
No of crystals in tyler mesh 20 = 1521
No of crystals in tyler mesh 14 = 1113
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.22429
cmwtfr in tyler mesh 35 = 97.57133
cmwtfr in tyler mesh 28 = 93.03802
cmwtfr in tyler mesh 20 = 81.85957
cmwtfr in tyler mesh 14 = 61.4355
Sandler A-test = .302526398842713
rms = 14.28759

time =37.0166666666667 hour
No of crystals in tyler mesh 65 = 10896
No of crystals in tyler mesh 48 = 2699
No of crystals in tyler mesh 35 = 2563
No of crystals in tyler mesh 28 = 2254
No of crystals in tyler mesh 20 = 1563
No of crystals in tyler mesh 14 = 1102
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.25972
cmwtfr in tyler mesh 35 = 97.5944
cmwtfr in tyler mesh 28 = 93.18676
cmwtfr in tyler mesh 20 = 82.33935
cmwtfr in tyler mesh 14 = 61.68658
Sandler A-test = .302473477023103
rms = 14.49697

time =38.0166666666667 hour
No of crystals in tyler mesh 65 = 10948
No of crystals in tyler mesh 48 = 2628
No of crystals in tyler mesh 35 = 2603
No of crystals in tyler mesh 28 = 2214
No of crystals in tyler mesh 20 = 1611
No of crystals in tyler mesh 14 = 1105
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.26936
cmwtfr in tyler mesh 35 = 97.68858
cmwtfr in tyler mesh 28 = 93.31765
cmwtfr in tyler mesh 20 = 82.97797
cmwtfr in tyler mesh 14 = 62.15949
Sandler A-test = .302955588809474
rms = 14.81103

time =39.0166666666667 hour
No of crystals in tyler mesh 65 = 10802
No of crystals in tyler mesh 48 = 2605
No of crystals in tyler mesh 35 = 2607
No of crystals in tyler mesh 28 = 2215

No of crystals in tyler mesh 20 = 1621
No of crystals in tyler mesh 14 = 1133
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.29192
cmwtfr in tyler mesh 35 = 97.76755
cmwtfr in tyler mesh 28 = 93.48762
cmwtfr in tyler mesh 20 = 83.35524
cmwtfr in tyler mesh 14 = 62.76393
Sandler A-test = .303550028311803
rms = 15.10743

time =40.0166666666667 hour
No of crystals in tyler mesh 65 = 10805
No of crystals in tyler mesh 48 = 2602
No of crystals in tyler mesh 35 = 2547
No of crystals in tyler mesh 28 = 2201
No of crystals in tyler mesh 20 = 1604
No of crystals in tyler mesh 14 = 1156
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.2887
cmwtfr in tyler mesh 35 = 97.77763
cmwtfr in tyler mesh 28 = 93.60469
cmwtfr in tyler mesh 20 = 83.72418
cmwtfr in tyler mesh 14 = 63.60073
Sandler A-test = .305635732914936
rms = 15.45935

time =41.0166666666667 hour
No of crystals in tyler mesh 65 = 10755
No of crystals in tyler mesh 48 = 2633
No of crystals in tyler mesh 35 = 2543
No of crystals in tyler mesh 28 = 2234
No of crystals in tyler mesh 20 = 1565
No of crystals in tyler mesh 14 = 1184
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.31725
cmwtfr in tyler mesh 35 = 97.84011
cmwtfr in tyler mesh 28 = 93.83126
cmwtfr in tyler mesh 20 = 84.08232
cmwtfr in tyler mesh 14 = 64.98148
Sandler A-test = .308270888559233
rms = 15.99199

time =42.0166666666667 hour
No of crystals in tyler mesh 65 = 10691
No of crystals in tyler mesh 48 = 2649
No of crystals in tyler mesh 35 = 2556
No of crystals in tyler mesh 28 = 2200
No of crystals in tyler mesh 20 = 1551
No of crystals in tyler mesh 14 = 1208
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33851
cmwtfr in tyler mesh 35 = 97.86464
cmwtfr in tyler mesh 28 = 93.88072
cmwtfr in tyler mesh 20 = 84.32304

cmwtfr in tyler mesh 14 = 65.71479
Sandler A-test = .30992519204701
rms = 16.27786

time =43.0166666666667 hour
No of crystals in tyler mesh 65 = 10728
No of crystals in tyler mesh 48 = 2623
No of crystals in tyler mesh 35 = 2600
No of crystals in tyler mesh 28 = 2227
No of crystals in tyler mesh 20 = 1522
No of crystals in tyler mesh 14 = 1199
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.35044
cmwtfr in tyler mesh 35 = 97.89392
cmwtfr in tyler mesh 28 = 93.88297
cmwtfr in tyler mesh 20 = 84.20128
cmwtfr in tyler mesh 14 = 65.94925
Sandler A-test = .31031778206736
rms = 16.32368

time =44.0166666666667 hour
No of crystals in tyler mesh 65 = 10941
No of crystals in tyler mesh 48 = 2556
No of crystals in tyler mesh 35 = 2607
No of crystals in tyler mesh 28 = 2181
No of crystals in tyler mesh 20 = 1532
No of crystals in tyler mesh 14 = 1203
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33035
cmwtfr in tyler mesh 35 = 97.93145
cmwtfr in tyler mesh 28 = 93.94152
cmwtfr in tyler mesh 20 = 84.67615
cmwtfr in tyler mesh 14 = 66.61213
Sandler A-test = .311983075230852
rms = 16.64419

time =44.5166666666667 hour
No of crystals in tyler mesh 65 = 10989
No of crystals in tyler mesh 48 = 2564
No of crystals in tyler mesh 35 = 2592
No of crystals in tyler mesh 28 = 2170
No of crystals in tyler mesh 20 = 1506
No of crystals in tyler mesh 14 = 1204
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.32237
cmwtfr in tyler mesh 35 = 97.9112
cmwtfr in tyler mesh 28 = 93.89577
cmwtfr in tyler mesh 20 = 84.62498
cmwtfr in tyler mesh 14 = 66.86152
Sandler A-test = .313084638674993
rms = 16.70193
----------steady state----------
time =45.0166666666667 hour
No of crystals in tyler mesh 65 = 11009
No of crystals in tyler mesh 48 = 2633

No of crystals in tyler mesh 35 = 2607
No of crystals in tyler mesh 28 = 2174
No of crystals in tyler mesh 20 = 1523
No of crystals in tyler mesh 14 = 1197
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33537
cmwtfr in tyler mesh 35 = 97.90461
cmwtfr in tyler mesh 28 = 93.87723
cmwtfr in tyler mesh 20 = 84.6563
cmwtfr in tyler mesh 14 = 66.82814
Sandler A-test = .312984021821086
rms = 16.69601

time =46.0166666666667 hour
No of crystals in tyler mesh 65 = 11028
No of crystals in tyler mesh 48 = 2608
No of crystals in tyler mesh 35 = 2548
No of crystals in tyler mesh 28 = 2241
No of crystals in tyler mesh 20 = 1497
No of crystals in tyler mesh 14 = 1162
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.32229
cmwtfr in tyler mesh 35 = 97.87884
cmwtfr in tyler mesh 28 = 93.95437
cmwtfr in tyler mesh 20 = 84.28938
cmwtfr in tyler mesh 14 = 66.33758
Sandler A-test = .311557978223198
rms = 16.47125

time =47.0166666666667 hour
No of crystals in tyler mesh 65 = 10966
No of crystals in tyler mesh 48 = 2623
No of crystals in tyler mesh 35 = 2547
No of crystals in tyler mesh 28 = 2182
No of crystals in tyler mesh 20 = 1525
No of crystals in tyler mesh 14 = 1159
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.31033
cmwtfr in tyler mesh 35 = 97.8652
cmwtfr in tyler mesh 28 = 93.92946
cmwtfr in tyler mesh 20 = 84.49644
cmwtfr in tyler mesh 14 = 66.42615
Sandler A-test = .312082695973558
rms = 16.54102

time =48.0166666666667 hour
No of crystals in tyler mesh 65 = 10878
No of crystals in tyler mesh 48 = 2673
No of crystals in tyler mesh 35 = 2575
No of crystals in tyler mesh 28 = 2170
No of crystals in tyler mesh 20 = 1524
No of crystals in tyler mesh 14 = 1143
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.31684
cmwtfr in tyler mesh 35 = 97.84576

cmwtfr in tyler mesh 28 = 93.84772
cmwtfr in tyler mesh 20 = 84.36368
cmwtfr in tyler mesh 14 = 66.01564
Sandler A-test = .311230930788754
rms = 16.37314

time =49.0166666666667 hour
No of crystals in tyler mesh 65 = 10851
No of crystals in tyler mesh 48 = 2677
No of crystals in tyler mesh 35 = 2572
No of crystals in tyler mesh 28 = 2172
No of crystals in tyler mesh 20 = 1538
No of crystals in tyler mesh 14 = 1155
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.34068
cmwtfr in tyler mesh 35 = 97.86553
cmwtfr in tyler mesh 28 = 93.91071
cmwtfr in tyler mesh 20 = 84.55933
cmwtfr in tyler mesh 14 = 66.33419
Sandler A-test = .311620897234251
rms = 16.52488

time =50.0166666666667 hour
No of crystals in tyler mesh 65 = 10957
No of crystals in tyler mesh 48 = 2642
No of crystals in tyler mesh 35 = 2604
No of crystals in tyler mesh 28 = 2171
No of crystals in tyler mesh 20 = 1553
No of crystals in tyler mesh 14 = 1134
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.32742
cmwtfr in tyler mesh 35 = 97.83487
cmwtfr in tyler mesh 28 = 93.78671
cmwtfr in tyler mesh 20 = 84.40119
cmwtfr in tyler mesh 14 = 65.83154
Sandler A-test = .310885896825434
rms = 16.31674

time =51.0166666666667 hour
No of crystals in tyler mesh 65 = 10952
No of crystals in tyler mesh 48 = 2590
No of crystals in tyler mesh 35 = 2652
No of crystals in tyler mesh 28 = 2121
No of crystals in tyler mesh 20 = 1535
No of crystals in tyler mesh 14 = 1129
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.32768
cmwtfr in tyler mesh 35 = 97.9052
cmwtfr in tyler mesh 28 = 93.77283
cmwtfr in tyler mesh 20 = 84.63011
cmwtfr in tyler mesh 14 = 66.17296
Sandler A-test = .311481348318927
rms = 16.47523

time =52.0166666666667 hour

No of crystals in tyler mesh 65 = 10923
No of crystals in tyler mesh 48 = 2635
No of crystals in tyler mesh 35 = 2557
No of crystals in tyler mesh 28 = 2152
No of crystals in tyler mesh 20 = 1472
No of crystals in tyler mesh 14 = 1159
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33311
cmwtfr in tyler mesh 35 = 97.87908
cmwtfr in tyler mesh 28 = 93.91793
cmwtfr in tyler mesh 20 = 84.79914
cmwtfr in tyler mesh 14 = 67.39142
Sandler A-test = .314688175776879
rms = 16.90607

time =53.0166666666667 hour
No of crystals in tyler mesh 65 = 10988
No of crystals in tyler mesh 48 = 2607
No of crystals in tyler mesh 35 = 2577
No of crystals in tyler mesh 28 = 2210
No of crystals in tyler mesh 20 = 1475
No of crystals in tyler mesh 14 = 1155
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.3308
cmwtfr in tyler mesh 35 = 97.92008
cmwtfr in tyler mesh 28 = 94.03738
cmwtfr in tyler mesh 20 = 84.79341
cmwtfr in tyler mesh 14 = 67.44263
Sandler A-test = .314166773420531
rms = 16.93865

time =54.0166666666667 hour
No of crystals in tyler mesh 65 = 10912
No of crystals in tyler mesh 48 = 2693
No of crystals in tyler mesh 35 = 2598
No of crystals in tyler mesh 28 = 2228
No of crystals in tyler mesh 20 = 1464
No of crystals in tyler mesh 14 = 1131
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33791
cmwtfr in tyler mesh 35 = 97.88251
cmwtfr in tyler mesh 28 = 93.92976
cmwtfr in tyler mesh 20 = 84.47625
cmwtfr in tyler mesh 14 = 67.18937
Sandler A-test = .314032777684063
rms = 16.77295

time =55.0166666666667 hour
No of crystals in tyler mesh 65 = 10860
No of crystals in tyler mesh 48 = 2706
No of crystals in tyler mesh 35 = 2634
No of crystals in tyler mesh 28 = 2227
No of crystals in tyler mesh 20 = 1497
No of crystals in tyler mesh 14 = 1131
cmwtfr in tyler mesh 65 = 99.99999

cmwtfr in tyler mesh 48 = 99.34525
cmwtfr in tyler mesh 35 = 97.8949
cmwtfr in tyler mesh 28 = 93.9264
cmwtfr in tyler mesh 20 = 84.57015
cmwtfr in tyler mesh 14 = 67.53493
Sandler A-test = .314934109786897
rms = 16.90144

time =56.0166666666667 hour
No of crystals in tyler mesh 65 = 10845
No of crystals in tyler mesh 48 = 2617
No of crystals in tyler mesh 35 = 2617
No of crystals in tyler mesh 28 = 2260
No of crystals in tyler mesh 20 = 1513
No of crystals in tyler mesh 14 = 1126
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.3266
cmwtfr in tyler mesh 35 = 97.90559
cmwtfr in tyler mesh 28 = 93.95223
cmwtfr in tyler mesh 20 = 84.40979
cmwtfr in tyler mesh 14 = 66.96484
Sandler A-test = .313226610976668
rms = 16.69275

time =57.0166666666667 hour
No of crystals in tyler mesh 65 = 10764
No of crystals in tyler mesh 48 = 2699
No of crystals in tyler mesh 35 = 2586
No of crystals in tyler mesh 28 = 2224
No of crystals in tyler mesh 20 = 1569
No of crystals in tyler mesh 14 = 1135
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.3584
cmwtfr in tyler mesh 35 = 97.93477
cmwtfr in tyler mesh 28 = 94.07123
cmwtfr in tyler mesh 20 = 84.8345
cmwtfr in tyler mesh 14 = 67.03375
Sandler A-test = .312509684702367
rms = 16.82736

time =58.0166666666667 hour
No of crystals in tyler mesh 65 = 10887
No of crystals in tyler mesh 48 = 2666
No of crystals in tyler mesh 35 = 2608
No of crystals in tyler mesh 28 = 2217
No of crystals in tyler mesh 20 = 1580
No of crystals in tyler mesh 14 = 1110
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.3609
cmwtfr in tyler mesh 35 = 97.92674
cmwtfr in tyler mesh 28 = 94.03017
cmwtfr in tyler mesh 20 = 84.79861
cmwtfr in tyler mesh 14 = 66.62698
Sandler A-test = .311494980242167
rms = 16.68854

time =59.0166666666667 hour
No of crystals in tyler mesh 65 = 11044
No of crystals in tyler mesh 48 = 2556
No of crystals in tyler mesh 35 = 2690
No of crystals in tyler mesh 28 = 2218
No of crystals in tyler mesh 20 = 1595
No of crystals in tyler mesh 14 = 1111
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.35316
cmwtfr in tyler mesh 35 = 97.99371
cmwtfr in tyler mesh 28 = 94.02881
cmwtfr in tyler mesh 20 = 84.87241
cmwtfr in tyler mesh 14 = 66.52699
Sandler A-test = .310841569763063
rms = 16.67824

time =60.0166666666667 hour
No of crystals in tyler mesh 65 = 11131
No of crystals in tyler mesh 48 = 2572
No of crystals in tyler mesh 35 = 2622
No of crystals in tyler mesh 28 = 2199
No of crystals in tyler mesh 20 = 1559
No of crystals in tyler mesh 14 = 1126
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33383
cmwtfr in tyler mesh 35 = 97.97628
cmwtfr in tyler mesh 28 = 94.0544
cmwtfr in tyler mesh 20 = 84.97659
cmwtfr in tyler mesh 14 = 67.02853
Sandler A-test = .312476957541544
rms = 16.85774

time =61.0166666666667 hour
No of crystals in tyler mesh 65 = 10984
No of crystals in tyler mesh 48 = 2730
No of crystals in tyler mesh 35 = 2534
No of crystals in tyler mesh 28 = 2226
No of crystals in tyler mesh 20 = 1530
No of crystals in tyler mesh 14 = 1145
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.34772
cmwtfr in tyler mesh 35 = 97.92094
cmwtfr in tyler mesh 28 = 94.1347
cmwtfr in tyler mesh 20 = 85.07236
cmwtfr in tyler mesh 14 = 67.63218
Sandler A-test = .314199178577868
rms = 17.07307

time =62.0166666666667 hour
No of crystals in tyler mesh 65 = 10910
No of crystals in tyler mesh 48 = 2751
No of crystals in tyler mesh 35 = 2528
No of crystals in tyler mesh 28 = 2261
No of crystals in tyler mesh 20 = 1501

No of crystals in tyler mesh 14 = 1120
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33391
cmwtfr in tyler mesh 35 = 97.84097
cmwtfr in tyler mesh 28 = 94.0424
cmwtfr in tyler mesh 20 = 84.57938
cmwtfr in tyler mesh 14 = 67.07618
Sandler A-test = .313578976711623
rms = 16.77387

time =63.0166666666667 hour
No of crystals in tyler mesh 65 = 10939
No of crystals in tyler mesh 48 = 2710
No of crystals in tyler mesh 35 = 2641
No of crystals in tyler mesh 28 = 2241
No of crystals in tyler mesh 20 = 1499
No of crystals in tyler mesh 14 = 1139
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.34596
cmwtfr in tyler mesh 35 = 97.90733
cmwtfr in tyler mesh 28 = 94.0383
cmwtfr in tyler mesh 20 = 84.73317
cmwtfr in tyler mesh 14 = 67.57117
Sandler A-test = .314519400969609
rms = 16.96455

time =64.0166666666667 hour
No of crystals in tyler mesh 65 = 10986
No of crystals in tyler mesh 48 = 2714
No of crystals in tyler mesh 35 = 2719
No of crystals in tyler mesh 28 = 2186
No of crystals in tyler mesh 20 = 1525
No of crystals in tyler mesh 14 = 1129
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.34044
cmwtfr in tyler mesh 35 = 97.88425
cmwtfr in tyler mesh 28 = 93.77081
cmwtfr in tyler mesh 20 = 84.47691
cmwtfr in tyler mesh 14 = 66.89059
Sandler A-test = .313664613604091
rms = 16.65959

time =65.0166666666667 hour
No of crystals in tyler mesh 65 = 11035
No of crystals in tyler mesh 48 = 2694
No of crystals in tyler mesh 35 = 2683
No of crystals in tyler mesh 28 = 2183
No of crystals in tyler mesh 20 = 1541
No of crystals in tyler mesh 14 = 1122
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.3227
cmwtfr in tyler mesh 35 = 97.8414
cmwtfr in tyler mesh 28 = 93.73537
cmwtfr in tyler mesh 20 = 84.48708
cmwtfr in tyler mesh 14 = 66.3995

Sandler A-test = .312745537533213
rms = 16.50344

time =66.01666666666667 hour
No of crystals in tyler mesh 65 = 11035
No of crystals in tyler mesh 48 = 2701
No of crystals in tyler mesh 35 = 2661
No of crystals in tyler mesh 28 = 2211
No of crystals in tyler mesh 20 = 1532
No of crystals in tyler mesh 14 = 1123
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33969
cmwtfr in tyler mesh 35 = 97.87347
cmwtfr in tyler mesh 28 = 93.8506
cmwtfr in tyler mesh 20 = 84.5909
cmwtfr in tyler mesh 14 = 66.55086
Sandler A-test = .31242753904466
rms = 16.59077

time =67.01666666666667 hour
No of crystals in tyler mesh 65 = 10976
No of crystals in tyler mesh 48 = 2618
No of crystals in tyler mesh 35 = 2709
No of crystals in tyler mesh 28 = 2236
No of crystals in tyler mesh 20 = 1497
No of crystals in tyler mesh 14 = 1135
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.3345
cmwtfr in tyler mesh 35 = 97.92648
cmwtfr in tyler mesh 28 = 93.85755
cmwtfr in tyler mesh 20 = 84.49099
cmwtfr in tyler mesh 14 = 66.84332
Sandler A-test = .312981143141941
rms = 16.66194

time =68.01666666666667 hour
No of crystals in tyler mesh 65 = 11019
No of crystals in tyler mesh 48 = 2642
No of crystals in tyler mesh 35 = 2667
No of crystals in tyler mesh 28 = 2262
No of crystals in tyler mesh 20 = 1455
No of crystals in tyler mesh 14 = 1148
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.34064
cmwtfr in tyler mesh 35 = 97.94502
cmwtfr in tyler mesh 28 = 93.96114
cmwtfr in tyler mesh 20 = 84.54438
cmwtfr in tyler mesh 14 = 67.743
Sandler A-test = .315162424959963
rms = 16.96774

time =69.01666666666667 hour
No of crystals in tyler mesh 65 = 11000
No of crystals in tyler mesh 48 = 2713
No of crystals in tyler mesh 35 = 2679

No of crystals in tyler mesh 28 = 2295
No of crystals in tyler mesh 20 = 1491
No of crystals in tyler mesh 14 = 1167
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.35182
cmwtfr in tyler mesh 35 = 97.94057
cmwtfr in tyler mesh 28 = 93.9974
cmwtfr in tyler mesh 20 = 84.71622
cmwtfr in tyler mesh 14 = 68.06862
Sandler A-test = .315916113364729
rms = 17.11202

time =70.0166666666667 hour
No of crystals in tyler mesh 65 = 10868
No of crystals in tyler mesh 48 = 2733
No of crystals in tyler mesh 35 = 2681
No of crystals in tyler mesh 28 = 2344
No of crystals in tyler mesh 20 = 1512
No of crystals in tyler mesh 14 = 1162
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.36913
cmwtfr in tyler mesh 35 = 97.94108
cmwtfr in tyler mesh 28 = 94.02653
cmwtfr in tyler mesh 20 = 84.56218
cmwtfr in tyler mesh 14 = 67.81499
Sandler A-test = .314935819447598
rms = 17.00281

time =71.0166666666667 hour
No of crystals in tyler mesh 65 = 10972
No of crystals in tyler mesh 48 = 2654
No of crystals in tyler mesh 35 = 2676
No of crystals in tyler mesh 28 = 2334
No of crystals in tyler mesh 20 = 1524
No of crystals in tyler mesh 14 = 1142
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.34394
cmwtfr in tyler mesh 35 = 97.93798
cmwtfr in tyler mesh 28 = 93.99815
cmwtfr in tyler mesh 20 = 84.34079
cmwtfr in tyler mesh 14 = 67.16167
Sandler A-test = .313322427357324
rms = 16.74637

time =72.0166666666667 hour
No of crystals in tyler mesh 65 = 10981
No of crystals in tyler mesh 48 = 2679
No of crystals in tyler mesh 35 = 2679
No of crystals in tyler mesh 28 = 2287
No of crystals in tyler mesh 20 = 1565
No of crystals in tyler mesh 14 = 1124
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.35983
cmwtfr in tyler mesh 35 = 97.97022
cmwtfr in tyler mesh 28 = 94.02898

cmwtfr in tyler mesh 20 = 84.71387
cmwtfr in tyler mesh 14 = 67.25992
Sandler A-test = .313100254803892
rms = 16.86613

time =73.0166666666667 hour
No of crystals in tyler mesh 65 = 10998
No of crystals in tyler mesh 48 = 2690
No of crystals in tyler mesh 35 = 2659
No of crystals in tyler mesh 28 = 2240
No of crystals in tyler mesh 20 = 1574
No of crystals in tyler mesh 14 = 1109
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.36318
cmwtfr in tyler mesh 35 = 97.95901
cmwtfr in tyler mesh 28 = 94.05848
cmwtfr in tyler mesh 20 = 84.95378
cmwtfr in tyler mesh 14 = 67.41994
Sandler A-test = .313485033799109
rms = 16.97286

time =74.0166666666667 hour
No of crystals in tyler mesh 65 = 10991
No of crystals in tyler mesh 48 = 2615
No of crystals in tyler mesh 35 = 2646
No of crystals in tyler mesh 28 = 2307
No of crystals in tyler mesh 20 = 1573
No of crystals in tyler mesh 14 = 1116
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.35324
cmwtfr in tyler mesh 35 = 97.98764
cmwtfr in tyler mesh 28 = 94.13931
cmwtfr in tyler mesh 20 = 84.81032
cmwtfr in tyler mesh 14 = 67.22337
Sandler A-test = .312534771647884
rms = 16.89222

time =75.0166666666667 hour
No of crystals in tyler mesh 65 = 10982
No of crystals in tyler mesh 48 = 2666
No of crystals in tyler mesh 35 = 2635
No of crystals in tyler mesh 28 = 2233
No of crystals in tyler mesh 20 = 1582
No of crystals in tyler mesh 14 = 1123
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.35835
cmwtfr in tyler mesh 35 = 97.98483
cmwtfr in tyler mesh 28 = 94.13574
cmwtfr in tyler mesh 20 = 85.07947
cmwtfr in tyler mesh 14 = 67.44095
Sandler A-test = .313135407011662
rms = 17.01934

time =76.0166666666667 hour
No of crystals in tyler mesh 65 = 10935

No of crystals in tyler mesh 48 = 2709
No of crystals in tyler mesh 35 = 2578
No of crystals in tyler mesh 28 = 2259
No of crystals in tyler mesh 20 = 1604
No of crystals in tyler mesh 14 = 1103
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.36588
cmwtfr in tyler mesh 35 = 97.95306
cmwtfr in tyler mesh 28 = 94.19566
cmwtfr in tyler mesh 20 = 85.05192
cmwtfr in tyler mesh 14 = 66.93319
Sandler A-test = .311600307073731
rms = 16.86372

time =76.5166666666667 hour
No of crystals in tyler mesh 65 = 10987
No of crystals in tyler mesh 48 = 2662
No of crystals in tyler mesh 35 = 2601
No of crystals in tyler mesh 28 = 2284
No of crystals in tyler mesh 20 = 1573
No of crystals in tyler mesh 14 = 1097
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.35903
cmwtfr in tyler mesh 35 = 97.95693
cmwtfr in tyler mesh 28 = 94.15465
cmwtfr in tyler mesh 20 = 84.76763
cmwtfr in tyler mesh 14 = 66.8354
Sandler A-test = .311495931219554
rms = 16.76381

Appendix III.3

Sample Program for Transient CSD in FC Crystallizer

Declaration

```
Dim wtfr(0 To 6) As Single
Dim cmwtfr(0 To 6), Ben(0 To 6) As Single
Dim wet(0 To 6) As Single
Dim cr(1 To 8000, 0 To 6) As Integer
Dim wt(1 To 8000, 0 To 6)  As Single
Dim tau, Tscan, t1, t2, nt, Num, size As Single
Dim top, bottom, count, A, rms As Single
Dim cryswt, Vplug, Vmixed, r, tp, tm As Single
```

Main Program

```
Sub main ()

Vplug = Val(mixed.Text1.Text)
tau = Val(mixed.Text2.Text) * 60
G = Val(mixed.Text3.Text) / 60
Num = Val(mixed.Text4.Text)

Open "sim_fc" For Output As #1
Print #1, "Results of transient MC Simulation"
Print #1, "PFR of total volume = " & Vplug * 100; " %"
Print #1, "Residence time in an imperfectly mixed crystallizer =" & tau / 60;
" hr"
Print #1, "Average growth rate of a crystal = " & G * 60; " mm/hr"
Print #1, "Compared with Run No. " & Num; " Bennett and Van Buren"

nt = 60
Print #1, "Nucleation rate =" & nt; " nuclei/min"

Ben(0) = 98.3
Ben(1) = 90.5
Ben(2) = 74.4
Ben(3) = 44.4
Ben(4) = 18.2
Ben(5) = 8.2
Ben(6) = 4.3
```

```
Vmixed = 1 - Vplug
r = Vplug / Vmixed
tp = tau * r / (r + 1)
tm = tau / (r + 1)

Tscan = 13 * tau
For t1 = 1 To Tscan
   For n = 1 To nt
      t2 = -tm * Log(Rnd) + tp
      ttot = t1 + t2
      If ttot > Tscan Then ttot = Tscan
      For times = t1 To ttot Step 1
         size = G * (times - t1)
         If size > .147 Then
            GoTo 250
         Else
            s = 0
            GoTo 310
         End If
250      If size > .208 Then
            GoTo 260
         Else
            s = 1
            GoTo 310
         End If
260      If size > .295 Then
            GoTo 270
         Else
            s = 2
            GoTo 310
         End If
270      If size > .417 Then
            GoTo 280
         Else
            s = 3
         GoTo 310
         End If
280      If size > .589 Then
            GoTo 290
         Else
            s = 4
            GoTo 310
         End If
290      If size < .833 Then
            s = 5
```

```
        Else
          s = 6
        End If
310     cr(times, s) = cr(times, s) + 1
        wt(times, s) = wt(times, s) + (size ^ 3)
      Next
      n5 = n5 + 1
      Debug.Print n5
      sumrnd = 0
    Next
  Next
  For times = 1 To Tscan Step 30
    For j = 0 To 6
      wet(j) = wt(times, j)
      cryswt = cryswt + wet(j)
    Next
    If cryswt = 0 Then
      GoTo 4
    Else
      wtfr(6) = (wet(6) / cryswt) * 100
      cmwtfr(6) = wtfr(6)
      For j = 5 To 0 Step -1
        wtfr(j) = (wet(j) / cryswt) * 100
        cmwtfr(j) = cmwtfr(j + 1) + wtfr(j)
      Next
      For j = 0 To 6
        top = top + (cmwtfr(j) - Ben(j)) ^ 2
        bottom = bottom + (cmwtfr(j) - Ben(j))
        count = count + 1
      Next
      A = top / bottom ^ 2
      rms = (top / count) ^ (1 / 2)
      Print #1, "time =" & times / 60; " hour"
      Print #1, " No of crystals in tyler mesh 100 = " & cr(times, 0)
      Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 1)
      Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 2)
      Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 3)
      Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 4)
      Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 5)
      Print #1, " No of crystals in tyler mesh 14 = " & cr(times, 6)
      Print #1, " cmwtfr in tyler mesh 100 = " & cmwtfr(0)
      Print #1, " cmwtfr in tyler mesh 65 = " & cmwtfr(1)
      Print #1, " cmwtfr in tyler mesh 48 = " & cmwtfr(2)
      Print #1, " cmwtfr in tyler mesh 35 = " & cmwtfr(3)
      Print #1, " cmwtfr in tyler mesh 28 = " & cmwtfr(4)
      Print #1, " cmwtfr in tyler mesh 20 = " & cmwtfr(5)
```

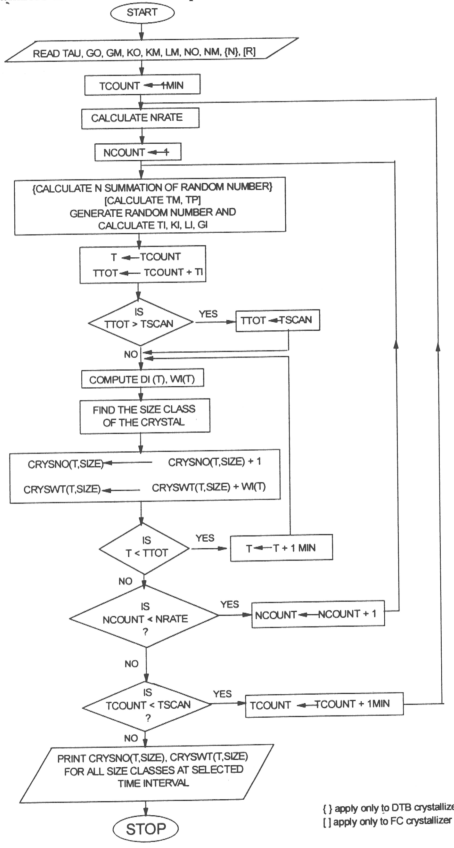
```
        Print #1, " cmwtfr in tyler mesh 14 = " & cmwtfr(6)
        Print #1, "Sandler A-test = " & A
        Print #1, "rms = " & rms
        Print #1,
        cryswt = 0
    End If
4  top = 0: bottom = 0: count = 0
Next

End Sub
```

APPENDIX IV

Appendix IV.1

Outline of Simulation Algorithm for Transient CSD in DTB and FC Crystallizers under Stochastic Dispersion Effects



Appendix IV.2

Sample Program for Transient CSD in DTB Crystallizer under Stochastic Dispersion Effects

Declaration

```
Dim wtfr(0 To 6) As Single
Dim cmwtfr(0 To 6) As Single
Dim wet(0 To 6) As Single
Dim cr(1 To 780, 0 To 6) As Integer
Dim wt(1 To 780, 0 To 6)  As Single
Dim tau, gmin, gmax, kmin, kmax, nmin, nmax, lm As Single
Dim t1 As Integer
```

Main Program

```
Sub main ()

nstage = Val(stage.Text1.Text)
Open "dtb" For Output As #1
Print #1, "Results of transient MC Simulation"
Print #1, "Number of stage=" & nstage
tau = 60
gmin = .001
gmax = .0015
kmin = .5238
kmax = 1
nmin = 50
nmax = 70
lm = .02
Print #1, "Tau=" & tau
Print #1, "Go=" & gmin
Print #1, "Gm=" & gmax
Print #1, "Ko=" & kmin
Print #1, "Km=" & kmax
Print #1, "No=" & nmin
Print #1, "Nm=" & nmax
Print #1, "Lm=" & lm

sdg = (gmax - gmin) / 6
```

```
sdk = (kmax - kmin) / 6
mg = (gmax + gmin) / 2
mk = (kmax + kmin) / 2
  For t1 = 1 To 780
    Randomize
    nt = nmin + (nmax - nmin) * Rnd
    For n = 1 To nt
      If nstage = 1 Then
        t2 = -tau * Log(Rnd)
      Else
        For st = 1 To nstage
          sumrnd = sumrnd + Log(Rnd)
        Next
        t2 = -(tau / nstage) * sumrnd
      End If
      tg = (-2 * (sdg ^ 2) * Log(Rnd)) ^ .5
      tk = (-2 * (sdk ^ 2) * Log(Rnd)) ^ .5
      x = Rnd
      If x < .5 Then
        g = mg - tg
        k = mk - tk
      Else
        g = mg + tg
        k = mk + tk
      End If
      ttot = t1 + t2
      If ttot > 780 Then ttot = 780
      l = 2 * lm * Rnd
      For times = t1 To ttot Step 1
        size = l + g * (times - t1)
        If size > .074 Then
          GoTo 260
        Else
          s = 0
          GoTo 310
        End If
260     If size > .147 Then
          GoTo 270
        Else
          s = 1
          GoTo 310
        End If
270     If size > .208 Then
          GoTo 280
        Else
          s = 2
```

```
            GoTo 310
          End If
280       If size > .295 Then
            GoTo 290
          Else
            s = 3
            GoTo 310
          End If
290       If size > .417 Then
            GoTo 300
          Else
            s = 4
            GoTo 310
          End If
300       If size < .589 Then
            s = 5
          Else
            s = 6
          End If
310       cr(times, s) = cr(times, s) + 1
          wt(times, s) = wt(times, s) + k * (size ^ 3)
        Next
        n5 = n5 + 1
        Debug.Print n5
        sumrnd = 0
      Next
    Next
    For times = 30 To 780 Step 30
      For j = 0 To 6
        wet(j) = wt(times, j)
        cryswt = cryswt + wet(j)
      Next
      wtfr(0) = wet(0) / cryswt
      cmwtfr(0) = wtfr(0)
      For j = 1 To 6
        wtfr(j) = wet(j) / cryswt
        cmwtfr(j) = cmwtfr(j - 1) + wtfr(j)
      Next
      Print #1, "time=" & times
      Print #1, " No of crystals in tyler mesh 200 = " & cr(times, 0)
      Print #1, " No of crystals in tyler mesh 100 = " & cr(times, 1)
      Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 2)
      Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 3)
      Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 4)
      Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 5)
      Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 6)
```

```
    Print #1, "cmwtfr in tyler mesh 200 = " & cmwtfr(0)
    Print #1, "cmwtfr in tyler mesh 100 = " & cmwtfr(1)
    Print #1, "cmwtfr in tyler mesh 65 = " & cmwtfr(2)
    Print #1, "cmwtfr in tyler mesh 48 = " & cmwtfr(3)
    Print #1, "cmwtfr in tyler mesh 35 = " & cmwtfr(4)
    Print #1, "cmwtfr in tyler mesh 28 = " & cmwtfr(5)
    Print #1, "cmwtfr in tyler mesh 20 = " & cmwtfr(6)
    cryswt = 0
  Next

End Sub
```

Appendix IV.3

Sample Program and Results for Transient CSD in FC Crystallizer under Stochastic Dispersion Effects

Declaration

```
Dim wtfr(0 To 6) As Single
Dim cmwtfr(0 To 6) As Single
Dim wet(0 To 6) As Single
Dim cr(1 To 780, 0 To 6) As Integer
Dim wt(1 To 780, 0 To 6)  As Single
Dim tau, gmin, gmax, kmin, kmax, nmin, nmax, lm As Single
Dim Vplug, Vmixed, r
Dim t1 As Integer
```

Main Program

```
Sub main ()

Vplug = Val(ratio.Text1.Text)
Open "FC" For Output As #1
Print #1, "Results of transient MC Simulation"
Print #1, "Ratio of PFR to Total Volume=" & Vplug
tau = 60
gmin = .001
gmax = .0015
kmin = .5238
kmax = 1
nmin = 50
nmax = 70
lm = .02
Print #1, "Tau=" & tau
Print #1, "Go=" & gmin
Print #1, "Gm=" & gmax
Print #1, "Ko=" & kmin
Print #1, "Km=" & kmax
Print #1, "No=" & nmin
Print #1, "Nm=" & nmax
Print #1, "Lm=" & lm
```

```
sdg = (gmax - gmin) / 6
sdk = (kmax - kmin) / 6
mg = (gmax + gmin) / 2
mk = (kmax + kmin) / 2
Vmixed = 1 - Vplug
r = Vplug / Vmixed
tp = tau * r / (r + 1)
tm = tau / (r + 1)
   For t1 = 1 To 780
      Randomize
      nt = nmin + (nmax - nmin) * Rnd
      For n = 1 To nt
         t2 = -tm * Log(Rnd) + tp
         tg = (-2 * (sdg ^ 2) * Log(Rnd)) ^ .5
         tk = (-2 * (sdk ^ 2) * Log(Rnd)) ^ .5
         x = Rnd
         If x < .5 Then
            g = mg - tg
            k = mk - tk
         Else
            g = mg + tg
            k = mk + tk
         End If
         ttot = t1 + t2
         If ttot > 780 Then ttot = 780
         l = 2 * lm * Rnd
         For times = t1 To ttot Step 1
            size = l + g * (times - t1)
            If size > .074 Then
               GoTo 260
            Else
               s = 0
               GoTo 310
            End If
260         If size > .147 Then
               GoTo 270
            Else
               s = 1
               GoTo 310
            End If
270         If size > .208 Then
               GoTo 280
            Else
               s = 2
               GoTo 310
            End If
```

```
280         If size > .295 Then
              GoTo 290
            Else
              s = 3
              GoTo 310
            End If
290         If size > .417 Then
              GoTo 300
            Else
              s = 4
              GoTo 310
            End If
300         If size < .589 Then
              s = 5
            Else
              s = 6
            End If
310         cr(times, s) = cr(times, s) + 1
            wt(times, s) = wt(times, s) + k * (size ^ 3)
          Next
          n5 = n5 + 1
          Debug.Print n5
        Next
      Next
      For times = 30 To 780 Step 30
        For j = 0 To 6
          wet(j) = wt(times, j)
          cryswt = cryswt + wet(j)
        Next
        wtfr(0) = wet(0) / cryswt
        cmwtfr(0) = wtfr(0)
        For j = 1 To 6
          wtfr(j) = wet(j) / cryswt
          cmwtfr(j) = cmwtfr(j - 1) + wtfr(j)
        Next
        Print #1, "time=" & times
        Print #1, " No of crystals in tyler mesh 200 = " & cr(times, 0)
        Print #1, " No of crystals in tyler mesh 100 = " & cr(times, 1)
        Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 2)
        Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 3)
        Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 4)
        Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 5)
        Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 6)
        Print #1, "cmwtfr in tyler mesh 200 = " & cmwtfr(0)
        Print #1, "cmwtfr in tyler mesh 100 = " & cmwtfr(1)
        Print #1, "cmwtfr in tyler mesh 65 = " & cmwtfr(2)
```

```
        Print #1, "cmwtfr in tyler mesh 48 = " & cmwtfr(3)
        Print #1, "cmwtfr in tyler mesh 35 = " & cmwtfr(4)
        Print #1, "cmwtfr in tyler mesh 28 = " & cmwtfr(5)
        Print #1, "cmwtfr in tyler mesh 20 = " & cmwtfr(6)
        cryswt = 0
    Next

End Sub
```

Sample of results (for 0% PFR)

```
Results of transient MC Simulation
Ratio of PFR to Total Volume=0
Tau=60
Go=.001
Gm=.0015
Ko=.5238
Km=1
No=50
Nm=70
Lm=.02

time=30
 No of crystals in tyler mesh 200 = 1389
 No of crystals in tyler mesh 100 = 3
 No of crystals in tyler mesh 65 = 0
 No of crystals in tyler mesh 48 = 0
 No of crystals in tyler mesh 35 = 0
 No of crystals in tyler mesh 28 = 0
 No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = .9861985
cmwtfr in tyler mesh 100 = 1
cmwtfr in tyler mesh 65 = 1
cmwtfr in tyler mesh 48 = 1
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=60
 No of crystals in tyler mesh 200 = 1838
 No of crystals in tyler mesh 100 = 433
 No of crystals in tyler mesh 65 = 0
 No of crystals in tyler mesh 48 = 0
 No of crystals in tyler mesh 35 = 0
 No of crystals in tyler mesh 28 = 0
 No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = .3889922
cmwtfr in tyler mesh 100 = 1
cmwtfr in tyler mesh 65 = 1
cmwtfr in tyler mesh 48 = 1
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
```

cmwtfr in tyler mesh 20 = 1

time=90
No of crystals in tyler mesh 200 = 1854
No of crystals in tyler mesh 100 = 936
No of crystals in tyler mesh 65 = 12
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = .1602252
cmwtfr in tyler mesh 100 = .9645727
cmwtfr in tyler mesh 65 = 1
cmwtfr in tyler mesh 48 = 1
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=120
No of crystals in tyler mesh 200 = 1794
No of crystals in tyler mesh 100 = 1086
No of crystals in tyler mesh 65 = 185
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 8.542478E-02
cmwtfr in tyler mesh 100 = .6421517
cmwtfr in tyler mesh 65 = 1
cmwtfr in tyler mesh 48 = 1
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=150
No of crystals in tyler mesh 200 = 1857
No of crystals in tyler mesh 100 = 1117
No of crystals in tyler mesh 65 = 348
No of crystals in tyler mesh 48 = 38
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 5.251395E-02
cmwtfr in tyler mesh 100 = .4159707
cmwtfr in tyler mesh 65 = .8849602
cmwtfr in tyler mesh 48 = 1
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=180
No of crystals in tyler mesh 200 = 1816
No of crystals in tyler mesh 100 = 1107
No of crystals in tyler mesh 65 = 393
No of crystals in tyler mesh 48 = 119

No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 3.958658E-02
cmwtfr in tyler mesh 100 = .3086667
cmwtfr in tyler mesh 65 = .7046975
cmwtfr in tyler mesh 48 = 1
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=210
No of crystals in tyler mesh 200 = 1807
No of crystals in tyler mesh 100 = 1072
No of crystals in tyler mesh 65 = 367
No of crystals in tyler mesh 48 = 194
No of crystals in tyler mesh 35 = 7
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 3.011664E-02
cmwtfr in tyler mesh 100 = .2339615
cmwtfr in tyler mesh 65 = .5366954
cmwtfr in tyler mesh 48 = .9641281
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=240
No of crystals in tyler mesh 200 = 1784
No of crystals in tyler mesh 100 = 1075
No of crystals in tyler mesh 65 = 375
No of crystals in tyler mesh 48 = 223
No of crystals in tyler mesh 35 = 38
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 2.469984E-02
cmwtfr in tyler mesh 100 = .1895512
cmwtfr in tyler mesh 65 = .4372618
cmwtfr in tyler mesh 48 = .8385384
cmwtfr in tyler mesh 35 = .9999999
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=270
No of crystals in tyler mesh 200 = 1787
No of crystals in tyler mesh 100 = 1056
No of crystals in tyler mesh 65 = 349
No of crystals in tyler mesh 48 = 224
No of crystals in tyler mesh 35 = 58
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 2.202494E-02
cmwtfr in tyler mesh 100 = .1752478
cmwtfr in tyler mesh 65 = .3820049
cmwtfr in tyler mesh 48 = .7646667

cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=300
No of crystals in tyler mesh 200 = 1850
No of crystals in tyler mesh 100 = 1036
No of crystals in tyler mesh 65 = 365
No of crystals in tyler mesh 48 = 215
No of crystals in tyler mesh 35 = 62
No of crystals in tyler mesh 28 = 1
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 2.210675E-02
cmwtfr in tyler mesh 100 = .1597621
cmwtfr in tyler mesh 65 = .3693035
cmwtfr in tyler mesh 48 = .7238129
cmwtfr in tyler mesh 35 = .9909256
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=330
No of crystals in tyler mesh 200 = 1849
No of crystals in tyler mesh 100 = 1093
No of crystals in tyler mesh 65 = 339
No of crystals in tyler mesh 48 = 196
No of crystals in tyler mesh 35 = 79
No of crystals in tyler mesh 28 = 5
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 2.050627E-02
cmwtfr in tyler mesh 100 = .1545391
cmwtfr in tyler mesh 65 = .3389382
cmwtfr in tyler mesh 48 = .6323854
cmwtfr in tyler mesh 35 = .9523973
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=360
No of crystals in tyler mesh 200 = 1923
No of crystals in tyler mesh 100 = 1049
No of crystals in tyler mesh 65 = 342
No of crystals in tyler mesh 48 = 199
No of crystals in tyler mesh 35 = 70
No of crystals in tyler mesh 28 = 5
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = .0225617
cmwtfr in tyler mesh 100 = .1645152
cmwtfr in tyler mesh 65 = .3465231
cmwtfr in tyler mesh 48 = .651548
cmwtfr in tyler mesh 35 = .9535317
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=390
No of crystals in tyler mesh 200 = 1800
No of crystals in tyler mesh 100 = 1076

No of crystals in tyler mesh 65 = 361
No of crystals in tyler mesh 48 = 163
No of crystals in tyler mesh 35 = 74
No of crystals in tyler mesh 28 = 10
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.998275E-02
cmwtfr in tyler mesh 100 = .1543041
cmwtfr in tyler mesh 65 = .3507973
cmwtfr in tyler mesh 48 = .5983913
cmwtfr in tyler mesh 35 = .8941116
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=420
No of crystals in tyler mesh 200 = 1811
No of crystals in tyler mesh 100 = 1054
No of crystals in tyler mesh 65 = 348
No of crystals in tyler mesh 48 = 182
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 9
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.968133E-02
cmwtfr in tyler mesh 100 = .1534835
cmwtfr in tyler mesh 65 = .3386047
cmwtfr in tyler mesh 48 = .5938872
cmwtfr in tyler mesh 35 = .900502
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=450
No of crystals in tyler mesh 200 = 1784
No of crystals in tyler mesh 100 = 1031
No of crystals in tyler mesh 65 = 364
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 78
No of crystals in tyler mesh 28 = 11
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.828448E-02
cmwtfr in tyler mesh 100 = .1385517
cmwtfr in tyler mesh 65 = .3177183
cmwtfr in tyler mesh 48 = .6078454
cmwtfr in tyler mesh 35 = .8979965
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=480
No of crystals in tyler mesh 200 = 1865
No of crystals in tyler mesh 100 = 1064
No of crystals in tyler mesh 65 = 352
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 19
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.688894E-02
cmwtfr in tyler mesh 100 = .1296194

No of crystals in tyler mesh 65 = 361
No of crystals in tyler mesh 48 = 163
No of crystals in tyler mesh 35 = 74
No of crystals in tyler mesh 28 = 10
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.998275E-02
cmwtfr in tyler mesh 100 = .1543041
cmwtfr in tyler mesh 65 = .3507973
cmwtfr in tyler mesh 48 = .5983913
cmwtfr in tyler mesh 35 = .8941116
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=420
No of crystals in tyler mesh 200 = 1811
No of crystals in tyler mesh 100 = 1054
No of crystals in tyler mesh 65 = 348
No of crystals in tyler mesh 48 = 182
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 9
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.968133E-02
cmwtfr in tyler mesh 100 = .1534835
cmwtfr in tyler mesh 65 = .3386047
cmwtfr in tyler mesh 48 = .5938872
cmwtfr in tyler mesh 35 = .900502
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=450
No of crystals in tyler mesh 200 = 1784
No of crystals in tyler mesh 100 = 1031
No of crystals in tyler mesh 65 = 364
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 78
No of crystals in tyler mesh 28 = 11
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.828448E-02
cmwtfr in tyler mesh 100 = .1385517
cmwtfr in tyler mesh 65 = .3177183
cmwtfr in tyler mesh 48 = .6078454
cmwtfr in tyler mesh 35 = .8979965
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=480
No of crystals in tyler mesh 200 = 1865
No of crystals in tyler mesh 100 = 1064
No of crystals in tyler mesh 65 = 352
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 19
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.688894E-02
cmwtfr in tyler mesh 100 = .1296194

cmwtfr in tyler mesh 65 = .2928881
cmwtfr in tyler mesh 48 = .5745622
cmwtfr in tyler mesh 35 = .8305749
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1
----------steady state----------
time=510
No of crystals in tyler mesh 200 = 1893
No of crystals in tyler mesh 100 = 1040
No of crystals in tyler mesh 65 = 339
No of crystals in tyler mesh 48 = 197
No of crystals in tyler mesh 35 = 85
No of crystals in tyler mesh 28 = 21
No of crystals in tyler mesh 20 = 1
cmwtfr in tyler mesh 200 = 1.724211E-02
cmwtfr in tyler mesh 100 = .1204618
cmwtfr in tyler mesh 65 = .2630246
cmwtfr in tyler mesh 48 = .5040336
cmwtfr in tyler mesh 35 = .7852369
cmwtfr in tyler mesh 28 = .9788637
cmwtfr in tyler mesh 20 = 1

time=540
No of crystals in tyler mesh 200 = 1867
No of crystals in tyler mesh 100 = 1049
No of crystals in tyler mesh 65 = 327
No of crystals in tyler mesh 48 = 180
No of crystals in tyler mesh 35 = 83
No of crystals in tyler mesh 28 = 24
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.734064E-02
cmwtfr in tyler mesh 100 = .1275765
cmwtfr in tyler mesh 65 = .2703285
cmwtfr in tyler mesh 48 = .4940478
cmwtfr in tyler mesh 35 = .7696309
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=570
No of crystals in tyler mesh 200 = 1889
No of crystals in tyler mesh 100 = 1033
No of crystals in tyler mesh 65 = 362
No of crystals in tyler mesh 48 = 181
No of crystals in tyler mesh 35 = 71
No of crystals in tyler mesh 28 = 18
No of crystals in tyler mesh 20 = 2
cmwtfr in tyler mesh 200 = 1.722681E-02
cmwtfr in tyler mesh 100 = .1254049
cmwtfr in tyler mesh 65 = .2885264
cmwtfr in tyler mesh 48 = .527334
cmwtfr in tyler mesh 35 = .7977287
cmwtfr in tyler mesh 28 = .9626451
cmwtfr in tyler mesh 20 = 1

time=600

No of crystals in tyler mesh 200 = 1886
No of crystals in tyler mesh 100 = 1071
No of crystals in tyler mesh 65 = 345
No of crystals in tyler mesh 48 = 181
No of crystals in tyler mesh 35 = 65
No of crystals in tyler mesh 28 = 18
No of crystals in tyler mesh 20 = 4
cmwtfr in tyler mesh 200 = .0174655
cmwtfr in tyler mesh 100 = .130412
cmwtfr in tyler mesh 65 = .2881138
cmwtfr in tyler mesh 48 = .5075626
cmwtfr in tyler mesh 35 = .7468447
cmwtfr in tyler mesh 28 = .9119091
cmwtfr in tyler mesh 20 = 1

time=630
No of crystals in tyler mesh 200 = 1855
No of crystals in tyler mesh 100 = 1076
No of crystals in tyler mesh 65 = 345
No of crystals in tyler mesh 48 = 194
No of crystals in tyler mesh 35 = 59
No of crystals in tyler mesh 28 = 21
No of crystals in tyler mesh 20 = 1
cmwtfr in tyler mesh 200 = .0187367
cmwtfr in tyler mesh 100 = .1355076
cmwtfr in tyler mesh 65 = .2966362
cmwtfr in tyler mesh 48 = .5446144
cmwtfr in tyler mesh 35 = .7540574
cmwtfr in tyler mesh 28 = .9681376
cmwtfr in tyler mesh 20 = 1

time=660
No of crystals in tyler mesh 200 = 1793
No of crystals in tyler mesh 100 = 1092
No of crystals in tyler mesh 65 = 369
No of crystals in tyler mesh 48 = 192
No of crystals in tyler mesh 35 = 64
No of crystals in tyler mesh 28 = 16
No of crystals in tyler mesh 20 = 2
cmwtfr in tyler mesh 200 = 1.685518E-02
cmwtfr in tyler mesh 100 = .1322738
cmwtfr in tyler mesh 65 = .2989801
cmwtfr in tyler mesh 48 = .548808
cmwtfr in tyler mesh 35 = .7744268
cmwtfr in tyler mesh 28 = .937535
cmwtfr in tyler mesh 20 = 1

time=690
No of crystals in tyler mesh 200 = 1842
No of crystals in tyler mesh 100 = 1048
No of crystals in tyler mesh 65 = 359
No of crystals in tyler mesh 48 = 195
No of crystals in tyler mesh 35 = 59
No of crystals in tyler mesh 28 = 15
No of crystals in tyler mesh 20 = 1

cmwtfr in tyler mesh 200 = .0178348
cmwtfr in tyler mesh 100 = .13652
cmwtfr in tyler mesh 65 = .3080125
cmwtfr in tyler mesh 48 = .5719891
cmwtfr in tyler mesh 35 = .7856463
cmwtfr in tyler mesh 28 = .9674219
cmwtfr in tyler mesh 20 = 1

time=720
No of crystals in tyler mesh 200 = 1828
No of crystals in tyler mesh 100 = 1051
No of crystals in tyler mesh 65 = 357
No of crystals in tyler mesh 48 = 188
No of crystals in tyler mesh 35 = 72
No of crystals in tyler mesh 28 = 10
No of crystals in tyler mesh 20 = 2
cmwtfr in tyler mesh 200 = 1.897358E-02
cmwtfr in tyler mesh 100 = .1421815
cmwtfr in tyler mesh 65 = .3144156
cmwtfr in tyler mesh 48 = .5710347
cmwtfr in tyler mesh 35 = .8349816
cmwtfr in tyler mesh 28 = .95425
cmwtfr in tyler mesh 20 = 1

time=750
No of crystals in tyler mesh 200 = 1799
No of crystals in tyler mesh 100 = 1053
No of crystals in tyler mesh 65 = 357
No of crystals in tyler mesh 48 = 186
No of crystals in tyler mesh 35 = 82
No of crystals in tyler mesh 28 = 12
No of crystals in tyler mesh 20 = 3
cmwtfr in tyler mesh 200 = 1.733694E-02
cmwtfr in tyler mesh 100 = .1313016
cmwtfr in tyler mesh 65 = .3010784
cmwtfr in tyler mesh 48 = .5420061
cmwtfr in tyler mesh 35 = .8360603
cmwtfr in tyler mesh 28 = .9313347
cmwtfr in tyler mesh 20 = 1

time=780
No of crystals in tyler mesh 200 = 1853
No of crystals in tyler mesh 100 = 1063
No of crystals in tyler mesh 65 = 339
No of crystals in tyler mesh 48 = 202
No of crystals in tyler mesh 35 = 70
No of crystals in tyler mesh 28 = 19
No of crystals in tyler mesh 20 = 2
cmwtfr in tyler mesh 200 = 1.752982E-02
cmwtfr in tyler mesh 100 = .1293155
cmwtfr in tyler mesh 65 = .2850104
cmwtfr in tyler mesh 48 = .5422521
cmwtfr in tyler mesh 35 = .7779839
cmwtfr in tyler mesh 28 = .9388199
cmwtfr in tyler mesh 20 = 1

iii) Dominant size (on a weight basis) occurs at $x = 3$

iv) Fractional breakage at $x = 3$ relative to product removal,

$$q = \frac{\beta_m 3^m y}{y} \text{ or } \beta_m = \frac{q}{3^m} \tag{AV.1.5}$$

v) Boundary conditions:

- For large x, $y(2x) << y(x)$; therefore suppress the birth term and consider only the death term.
- $y(0) = 1$

the following differential equation results:

$$\dot{y} + (1 + \beta_m x^m) y = g(x) \tag{AV.1.6}$$

where $$g(x) = 2^{m+1} \beta_m x^m \exp - \left[2x + \frac{\beta_m}{m+1} (2x)^{m+1} \right] \tag{AV.1.7}$$

Cumulative and differential weight distributions can be obtained from Eq. (AV.1.6) using the transformations

$$w = \int_0^x \dot{p}\, y dp \tag{AV.1.8}$$

$$\dot{w} = x^3 y \tag{AV.1.9}$$

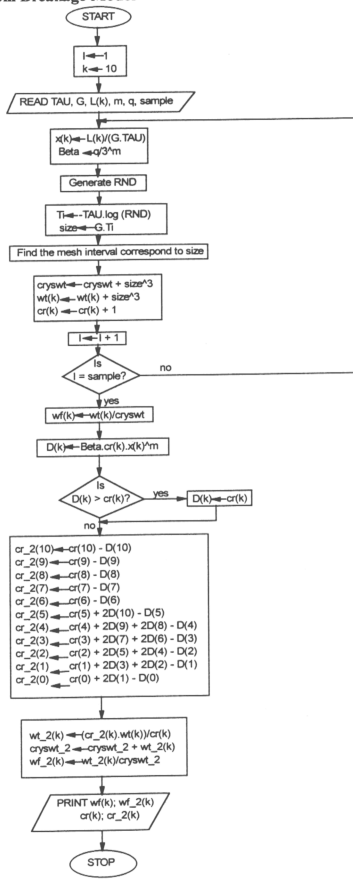
$$\ddot{w} = 3x^2 y + x^3 \dot{y} \tag{AV.1.10}$$

resulting in the final second-order differential equation for cumulative weight distribution,

$$\dot{w} = \left[\frac{3}{x} - 1 - \beta_m x^m\right] \dot{w} + x^3 g(x) \tag{AV.1.11}$$

Appendix V.2

Outline of Simulation Algorithm for Steady State CSD with Random Breakage Model



Appendix V.3

Sample Program and Results for Steady State CSD with Random Breakage Model

Declaration

```
Dim wtfr(0 To 10), wtfr_2(0 To 10) As Single
Dim wt(0 To 10), wt_2(0 To 10) As Single
Dim cr(0 To 10), cr_2(0 To 10) As Single
Dim cum(0 To 10), cum_2(0 To 10) As Single
Dim L(0 To 11), D(0 To 10) As Single
Dim x(0 To 10), cons(0 To 10) As Single
Dim sample, q, m, g, tau, ti, size, tem, tem_2 As Single
Dim cryswt, cryswt_2, crysno, crysno_2 As Single
Dim k As Integer
```

Main Program

```
Sub main ()

sample = Val(randolph.Text1.Text)
q = Val(randolph.Text2.Text)
m = Val(randolph.Text3.Text)
g = .00125
tau = 60

Open "randolph" For Output As #1
Print #1, "Results of steady state MC Simulation"
Print #1, "Sample size at steady state=" & sample
Print #1, "Breakage fraction=" & q
Print #1, "m=" & m
Print #1, "Growth rate=" & g; " mm/min"
Print #1, "Average residence time of a crystal in the crystallizer=" & tau; "
min"
Print #1,
L(0) = 0
L(1) = .075
L(2) = .15
L(3) = .225
L(4) = .3
```

```
L(5) = .375
L(6) = .45
L(7) = .525
L(8) = .6
L(9) = .675
L(10) = .75
L(11) = .825

For i = 1 To sample
    Randomize
    ti = -tau * Log(Rnd)
    size = g * ti
    If size > .075 Then
       GoTo 250
    Else
       k = 0
       GoTo 410
    End If
250 If size > .15 Then
       GoTo 260
    Else
       k = 1
       GoTo 410
    End If
260 If size > .225 Then
       GoTo 270
    Else
       k = 2
       GoTo 410
    End If
270 If size > .3 Then
       GoTo 280
    Else
       k = 3
       GoTo 410
    End If
280 If size > .375 Then
       GoTo 290
    Else
       k = 4
       GoTo 410
    End If
290 If size > .45 Then
       GoTo 300
    Else
       k = 5
```

```
        GoTo 410
    End If
300 If size > .525 Then
        GoTo 310
    Else
        k = 6
        GoTo 410
    End If
310 If size > .6 Then
        GoTo 320
    Else
        k = 7
        GoTo 410
    End If
320 If size > .675 Then
        GoTo 330
    Else
        k = 8
        GoTo 410
    End If
330 If size > .75 Then
        GoTo 340
    Else
        k = 9
        GoTo 410
    End If
340 k = 10
410 cr(k) = cr(k) + 1
    wt(k) = wt(k) + (size ^ 3)
Next

For k = 0 To 10
  crysno = crysno + cr(k)
  cryswt = cryswt + wt(k)
Next

For k = 0 To 10
  x(k) = L(k) / (g * tau)
Next

For k = 0 To 10
  cons(k) = (q / (3 ^ m)) * x(k) ^ m
Next

Rem cons(5) to cons(10) is greater than 1 which result in
Rem the death particle, D(5) to D(10), greater than its
```

```
Rem initial particle number, cr(5) to cr(10), if Randolph
Rem Breakage Model is to apply. "Number imbalance!!"
Rem Therefore:

For k = 5 To 10
   D(k) = cr(k)
Next

Rem Assume non-multiple breakage
Rem D(4) = cons(4) * (cr(4) + 2 * D(9) + 2 * D(8))
Rem D(3) = cons(3) * (cr(3) + 2 * D(7) + 2 * D(6))
Rem D(2) = cons(2) * (cr(2) + 2 * D(5) + 2 * D(4))
Rem D(1) = cons(1) * (cr(1) + 2 * D(3) + 2 * D(2))
Rem D(0) = 0

D(4) = cons(4) * cr(4)
D(3) = cons(3) * cr(3)
D(2) = cons(2) * cr(2)
D(1) = cons(1) * cr(1)
D(0) = 0

cr_2(10) = cr(10) - D(10)
cr_2(9) = cr(9) - D(9)
cr_2(8) = cr(8) - D(8)
cr_2(7) = cr(7) - D(7)
cr_2(6) = cr(6) - D(6)
cr_2(5) = cr(5) + 2 * D(10) - D(5)
cr_2(4) = cr(4) + 2 * D(9) + 2 * D(8) - D(4)
cr_2(3) = cr(3) + 2 * D(7) + 2 * D(6) - D(3)
cr_2(2) = cr(2) + 2 * D(5) + 2 * D(4) - D(2)
cr_2(1) = cr(1) + 2 * D(3) + 2 * D(2) - D(1)
cr_2(0) = cr(0) + 2 * D(1) - D(0)

For k = 0 To 10
   wt_2(k) = (cr_2(k) / cr(k)) * wt(k)
Next

For k = 0 To 10
   crysno_2 = crysno_2 + cr_2(k)
   cryswt_2 = cryswt_2 + wt_2(k)
Next

For k = 0 To 10
   wtfr(k) = (wt(k) / cryswt) * 100
   wtfr_2(k) = (wt_2(k) / cryswt_2) * 100
Next
```

```
tem = 0
tem_2 = 0
For k = 0 To 10
   cum_2(k) = tem_2 + wtfr_2(k)
   tem_2 = cum_2(k)
   cum(k) = tem + wtfr(k)
   tem = cum(k)
Next

Print #1, "Total number of crystals = " & crysno
For k = 0 To 10
   Print #1, "For x ("; k + 1; "), mass fraction under-size ="; wtfr(k)
   Print #1, "              cumulative mass fraction ="; cum(k)
   Print #1, "NO. OF CRYSTALS ="; cr(k)
Next

Print #1,
Print #1, "Total number of crystals including breakage = " & crysno_2
For k = 0 To 10
   Print #1, "For x ("; k + 1; "), mass fraction undersize ="; wtfr_2(k)
   Print #1, "              cumulative mass fraction ="; cum_2(k)
   Print #1, "NO. OF CRYSTALS ="; cr_2(k)
Next

End Sub
```

Sample of results (for m=4 and q=0.2)

```
Results of steady state MC Simulation
Sample size at steady state=100000
Breakage fraction=.2
m=4
Growth rate=.00125 mm/min
Average residence time of a crystal in the crystallizer=60 min

Total number of crystals = 100000
For x ( 1 ), mass fraction under-size = 1.87917693212762
          cumulative mass fraction = 1.87917693212762
NO. OF CRYSTALS = 63271
For x ( 2 ), mass fraction under-size = 12.2246079490638
          cumulative mass fraction = 14.1037848811914
NO. OF CRYSTALS = 23173
For x ( 3 ), mass fraction under-size = 21.0311248412267
          cumulative mass fraction = 35.1349097224182
NO. OF CRYSTALS = 8647
For x ( 4 ), mass fraction under-size = 20.3870342264851
          cumulative mass fraction = 55.5219439489032
```

NO. OF CRYSTALS = 3023
For x (5), mass fraction under-size = 17.0891888560525
cumulative mass fraction = 72.6111328049557
NO. OF CRYSTALS = 1191
For x (6), mass fraction under-size = 11.401246711006
cumulative mass fraction = 84.0123795159617
NO. OF CRYSTALS = 428
For x (7), mass fraction under-size = 7.28952908359445
cumulative mass fraction = 91.3019085995562
NO. OF CRYSTALS = 169
For x (8), mass fraction under-size = 4.0796034250265
cumulative mass fraction = 95.3815120245827
NO. OF CRYSTALS = 60
For x (9), mass fraction under-size = 2.51863625795787
cumulative mass fraction = 97.9001482825406
NO. OF CRYSTALS = 25
For x (10), mass fraction under-size = 1.06700049093263
cumulative mass fraction = 98.9671487734732
NO. OF CRYSTALS = 8
For x (11), mass fraction under-size = 1.0328512265268
cumulative mass fraction = 100
NO. OF CRYSTALS = 5

Total number of crystals including breakage = 102451.3
For x (1), mass fraction undersize = 2.771816
cumulative mass fraction = 2.771816
NO. OF CRYSTALS = 63385.43
For x (2), mass fraction undersize = 19.42438
cumulative mass fraction = 22.1962
NO. OF CRYSTALS = 25008.2
For x (3), mass fraction undersize = 38.19913
cumulative mass fraction = 60.39532
NO. OF CRYSTALS = 10667.05
For x (4), mass fraction undersize = 28.56125
cumulative mass fraction = 88.95657
NO. OF CRYSTALS = 2876.4
For x (5), mass fraction undersize = 10.65121
cumulative mass fraction = 99.60779
NO. OF CRYSTALS = 504.1703
For x (6), mass fraction undersize = .3922118
cumulative mass fraction = 100
NO. OF CRYSTALS = 10
For x (7), mass fraction undersize = 0
cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (8), mass fraction undersize = 0
cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (9), mass fraction undersize = 0
cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (10), mass fraction undersize = 0
cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (11), mass fraction undersize = 0

cumulative mass fraction = 100
NO. OF CRYSTALS = 0

APPENDIX VI

Appendix VI.1

Experimental Procedure

1. Prepare a saturated solution of brine by heating a saturated solution along with excess NaCl and subsequently cooling it. Use granular NaCl[^] (99.5% purity).
2. Dry the grinned granular sodium chloride[^] at 40°C for 24hours.
3. Sieve it at room temperature and note the screen fraction. Select the sieved salt with size fraction between 1400 to 1800μm.
4. Reconstitute the original NaCl by mixing the selected size fractions.
5. Fill the mixing rig with saturated NaCl solution (from step 1) and add the reconstitute NaCl to achieve 10% slurry.
6. Control the stirrer rpm at 200rpm.
7. Take a sample at an interval of 15min.
8. Allow the stirrer to run for 1hour.
9. Repeat the run (steps 7 and 8) for different rpm, namely 300, 400 and 500rpm.

[^] Source: J. T. Baker Inc. (CAS No. 7647-14-5; MERCK Index: 11, 8544)

Appendix VI.2

Sample Program and Results for Simulation of the Experimental Results Generated from the Coulter Counter -LS Particle Size Analyzer

Declaration

```
Dim L(1 To 92), y(1 To 92), yw(1 To 92) As Single
Dim ywf(1 To 92), ywb(1 To 92), ywc(1 To 92) As Single
Dim xb(1 To 92), xf(1 To 92), xc(1 To 92) As Single
Dim rpm, t, Lc, Lf, Lmax, top, bottom, statis As Single
Dim count, rms, tem As Single
Dim i, j As Single
```

Main Program

```
Sub main ()

rpm = (Coulter.Text1.Text)
t = Val(Coulter.Text2.Text)
statis = Val(Coulter.Text3.Text)
Lf = 200
Lc = 500
Lmax = 1822
top = 0
bottom = 0
count = 0
tem = 1000

Open "exp2" For Output As #1
Print #1, "rpm=" & rpm
Print #1, "Duration=" & t; " hr"
Print #1, "Lf=" & Lf; " mu"
Print #1, "Lc=" & Lc; " mu"
Print #1, "Lmax=" & Lmax; " mu"

L(1) = .375: y(1) = 0
L(2) = .412: y(2) = 0
L(3) = .452: y(3) = 0
L(4) = .496: y(4) = 0
L(5) = .545: y(5) = 0
```

L(6) = .598: y(6) = 0
L(7) = .657: y(7) = 0
L(8) = .721: y(8) = 0
L(9) = .791: y(9) = 0
L(10) = .869: y(10) = 0
L(11) = .953: y(11) = 0
L(12) = 1.047: y(12) = 0
L(13) = 1.149: y(13) = 0
L(14) = 1.261: y(14) = 0
L(15) = 1.385: y(15) = 0
L(16) = 1.52: y(16) = 0
L(17) = 1.669: y(17) = 0
L(18) = 1.832: y(18) = 0
L(19) = 2.01: y(19) = 0
L(20) = 2.207: y(20) = 0
L(21) = 2.423: y(21) = 0
L(22) = 2.66: y(22) = 0
L(23) = 2.92: y(23) = 0
L(24) = 3.206: y(24) = 0
L(25) = 3.519: y(25) = 0
L(26) = 3.862: y(26) = 0
L(27) = 4.241: y(27) = 0
L(28) = 4.656: y(28) = 0
L(29) = 5.111: y(29) = 0
L(30) = 5.611: y(30) = 0
L(31) = 6.158: y(31) = 0
L(32) = 6.761: y(32) = 0
L(33) = 7.421: y(33) = 0
L(34) = 8.147: y(34) = 0
L(35) = 8.944: y(35) = 0
L(36) = 9.819: y(36) = 0
L(37) = 10.78: y(37) = 0
L(38) = 11.83: y(38) = 0
L(39) = 12.99: y(39) = 0
L(40) = 14.26: y(40) = 0
L(41) = 15.65: y(41) = 0
L(42) = 17.18: y(42) = 0
L(43) = 18.86: y(43) = 0
L(44) = 20.7: y(44) = 0
L(45) = 22.73: y(45) = 0
L(46) = 24.95: y(46) = 0
L(47) = 27.38: y(47) = 0
L(48) = 30.07: y(48) = 0
L(49) = 33: y(49) = 0
L(50) = 36.24: y(50) = 0
L(51) = 39.77: y(51) = 0

```
L(52) = 43.66: y(52) = 0
L(53) = 47.93: y(53) = 0
L(54) = 52.63: y(54) = 0
L(55) = 57.77: y(55) = 0
L(56) = 63.41: y(56) = 0
L(57) = 69.62: y(57) = 0
L(58) = 76.43: y(58) = 0
L(59) = 83.9: y(59) = 0
L(60) = 92.09: y(60) = 0
L(61) = 101.1: y(61) = 0
L(62) = 111: y(62) = 0
L(63) = 121.8: y(63) = 0
L(64) = 133.7: y(64) = .0004
L(65) = 146.8: y(65) = .00304
L(66) = 161.2: y(66) = .023
L(67) = 176.8: y(67) = .071
L(68) = 194.2: y(68) = .136
L(69) = 213.2: y(69) = .212
L(70) = 234.1: y(70) = .296
L(71) = 256.8: y(71) = .394
L(72) = 282.1: y(72) = .518
L(73) = 309.6: y(73) = .679
L(74) = 339.8: y(74) = .882
L(75) = 373.1: y(75) = 1.128
L(76) = 409.6: y(76) = 1.431
L(77) = 449.7: y(77) = 1.83
L(78) = 493.6: y(78) = 2.409
L(79) = 541.9: y(79) = 3.309
L(80) = 594.9: y(80) = 4.719
L(81) = 653: y(81) = 6.889
L(82) = 716.9: y(82) = 10.049
L(83) = 786.9: y(83) = 14.409
L(84) = 863.9: y(84) = 20.099
L(85) = 948.2: y(85) = 27.139
L(86) = 1041: y(86) = 35.449
L(87) = 1143: y(87) = 44.839
L(88) = 1255: y(88) = 55.089
L(89) = 1377: y(89) = 65.939
L(90) = 1512: y(90) = 77.179
L(91) = 1660: y(91) = 88.639
L(92) = 1822: y(92) = 99.999

For kf = .001 To .1 Step .001
  For kb = .001 To .1 Step .001
    For i = 1 To 92
      If y(i) > 0 And kb > kf Then
```

Sample of results (for 1 hour duration and 200 rpm)

rpm=200
Duration=1 hr
Lf=200 mu
Lc=500 mu
Lmax=1822 mu

A= .161762236413652
rms= 8.0730543425311
kb= 7.30000000000001E-02
kf= .001

133.7	.0004	.8691363
146.8	.00304	.9578856
161.2	.023	1.055723
176.8	.071	1.161506
194.2	.136	1.278437
213.2	.212	1.392734
234.1	.296	1.511508
256.8	.394	1.638977
282.1	.518	1.779188
309.6	.679	1.929399
339.8	.882	2.091766
373.1	1.128	2.267709
409.6	1.431	2.4569
449.7	1.83	2.660428
493.6	2.409	2.878166
541.9	3.309	4.252327
594.9	4.719	5.014733
653	6.889	6.020637
716.9	10.049	7.373699
786.9	14.409	9.209448
863.9	20.099	11.73254
948.2	27.139	15.19916
1041	35.449	19.98682
1143	44.839	26.55108
1255	55.089	35.42357
1377	65.939	47.04337
1512	77.179	61.96422
1660	88.639	79.9642
1822	99.999	100

Sample of results (for 1 hour duration and 300 rpm)

rpm=300
Duration=1 hr
Lf=200 mu
Lc=500 mu
Lmax=1822 mu

A= .161942682739573
rms= 7.51406326523715

kb= 7.40000000000001E-02
kf= .001

133.7	.02	.8579433
146.8	.071	.9456395
161.2	.142	1.042328
176.8	.219	1.146876
194.2	.291	1.262443
213.2	.353	1.375264
234.1	.413	1.492434
256.8	.487	1.618181
282.1	.596	1.756496
309.6	.756	1.904677
339.8	.969	2.064851
373.1	1.226	2.238415
409.6	1.532	2.42505
449.7	1.917	2.625828
493.6	2.458	2.840624
541.9	3.29	4.212861
594.9	4.6	4.972453
653	6.63	5.975576
716.9	9.61	7.325986
786.9	13.77	9.159375
863.9	19.24	11.68061
948.2	26.07	15.14616
1041	34.21	19.93395
1143	43.5	26.50006
1255	53.76	35.3767
1377	64.76	47.0035
1512	76.28	61.93476
1660	88.15	79.94838
1822	100	100

Sample of results (for 1 hour duration and 400 rpm)

rpm=400
Duration=1 hr
Lf=200 mu
Lc=500 mu
Lmax=1822 mu

A= .161380377769634
rms= 9.99819373084631
kb= 9.70000000000001E-02
kf= .016

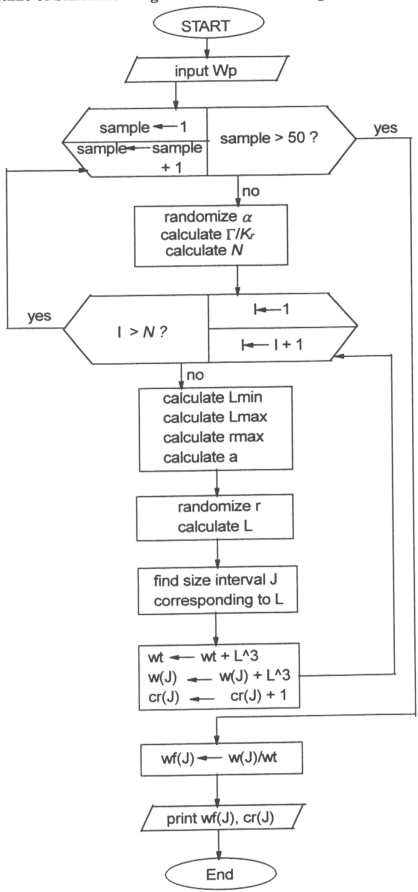
83.9	.014	.8707259
92.09	.044	.9917412
101.1	.072	1.135004
111	.087	1.304698
121.8	.117	1.504022
133.7	.2	1.739401
146.8	.325	2.014607
161.2	.434	2.331078
176.8	.532	2.68169

194.2	.701	3.06933
213.2	.935	3.294529
234.1	1.076	3.45022
256.8	1.083	3.617308
282.1	1.195	3.801098
309.6	1.632	3.997996
339.8	2.289	4.210829
373.1	2.904	4.441456
409.6	3.491	4.68945
449.7	4.337	4.956236
493.6	5.667	5.241651
541.9	7.457	6.641553
594.9	9.667	7.460359
653	12.547	8.518214
716.9	16.397	9.9149
786.9	21.257	11.77999
863.9	27.157	14.31012
948.2	34.297	17.75025
1041	42.937	22.46279
1143	52.717	28.88406
1255	62.997	37.5232
1377	73.257	48.79963
1512	82.937	63.24569
1660	91.647	80.64637
1822	99.997	100

APPENDIX VII

Appendix VII.1

Outline of Simulation Algorithm for CSD of Fragments due to Attrition



Appendix VII.2

Sample Program and Results for CSD of Fragments due to Attrition

Substances = MS

Declaration

```
Dim x(1 To 9) As Single
Dim cr(1 To 8), q(1 To 8) As Single
Dim wt(1 To 8), wtfr(1 To 8) As Single
Dim substances, Hv, m, Wc, Wp, ratio As Single
Dim i, j As Single
Dim FracR, N As Single
Dim Amin, Amak, Aa, Ak, A, ran1 As Single
Dim Lmin, Lmak, L, rmin, rmak, r, slope, cons, konst, Va As Single
Dim crysno, cryswt As Single
Dim xm, ym, xy, xmean, xsqrt, ymean, top, bottom As Single
Dim k, count As Integer
```

Main Program

```
Sub main ()

substances = (mersmann.Text1.Text)
Hv = Val(mersmann.Text2.Text) * 1000000
m = Val(mersmann.Text3.Text) * 1000000000
Wc = Val(mersmann.Text4.Text) * .0000000001
Wp = Val(mersmann.Text5.Text) * .000001
ratio = Val(mersmann.Text6.Text)


Open "mer" For Output As #1
Print #1, "Substances=" & substances
Print #1, "Vicker Hardness=" & Hv; " Pa"
Print #1, "Quasi-isotropic Shear Modulus=" & m; " Pa"
Print #1, "Critical Work to form cracks=" & Wc; " J"
Print #1, "Impact Energy=" & Wp; " J"
Print #1, "Ratio of Efficiency Constant=" & ratio

Amin = .5236
```

```
Amak = 1
Aa = (Amak + Amin) / 2
Ab = (Amak - Amin) / 6

FracR = (ratio * Wc ^ (1 / 3) * Hv ^ (5 / 3)) / (5.2 * m)
Lmin = (32 * m * FracR) / (3 * Hv ^ 2)
rmin = (((Lmin * Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (3 * m * FracR)) ^ (1 / 4))
rmax = ((Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (2 * 3 * m * FracR)) ^ (1 / 3)
Lmak = rmax / 2
konst = ((2 / 3) * (m ^ 3) * (FracR ^ 3)) / ((.0007 ^ (4 / 3)) * (Hv ^ 6))
cons = 13 * Log(rmin)
x(1) = Lmin
x(9) = Lmak
Rem For i = 2 To 9
Rem   x(i) = x(i - 1) + (Lmak - Lmin) / 9
Rem Next
x(2) = .000003
x(3) = .000005
x(4) = .00001
x(5) = .000015
x(6) = .00002
x(7) = .000025
x(8) = .00003

For i = 1 To 50
   Debug.Print i
   Randomize
   Ak = (-2 * (Ab ^ 2) * Log(Rnd)) ^ .5
   rand1 = Rnd
   If rand1 < .5 Then
      A = Aa - Ak
   Else
      A = Aa + Ak
   End If
   N = (.0007 * Wp * (Hv ^ 5)) / (A * m ^ 3 * FracR ^ 3)
   Debug.Print N
   For j = 1 To N
      Randomize
200  r = Exp((cons - Log(Rnd)) / 13)
      L = (3 * m * FracR * r ^ 4) / (Wp ^ (4 / 3) * Hv ^ (2 / 3))
      If L > x(1) Then
         GoTo 250
      Else
         GoTo 200
      End If
250  If L > x(2) Then
```

```
        GoTo 260
    Else
        k = 1
        GoTo 1000
    End If
260 If L > x(3) Then
        GoTo 270
    Else
        k = 2
        GoTo 1000
    End If
270 If L > x(4) Then
        GoTo 280
    Else
        k = 3
        GoTo 1000
    End If
280 If L > x(5) Then
        GoTo 290
    Else
        k = 4
        GoTo 1000
    End If
290 If L > x(6) Then
        GoTo 300
    Else
        k = 5
        GoTo 1000
    End If
300 If L > x(7) Then
        GoTo 310
    Else
        k = 6
    GoTo 1000
    End If
310 If L > x(8) Then
        GoTo 320
    Else
        k = 7
        GoTo 1000
    End If
320 If L > x(9) Then
        GoTo 200
    Else
        k = 8
        GoTo 1000
```

```
        End If

    1000cr(k) = cr(k) + 1
        wt(k) = wt(k) + L ^ 3
      Next
      Va = Va + konst * (N * A) ^ (4 / 3)
    Next

    For k = 1 To 8
      crysno = crysno + cr(k)
      cryswt = cryswt + wt(k)
    Next

    For k = 1 To 8
      q(k) = (cr(k) / crysno) * 100
      wtfr(k) = (wt(k) / cryswt) * 100
    Next

    Print #1, "Total number of fragments for 50 samples =" & crysno
    Print #1, "Total volume removed by attrition per sample =" & Va / 50; " m3"
    Print #1, "Lmin =" & x(1); " rmin =" & rmin
    Print #1, "Lmax =" & x(9); " rmax =" & rmax
    Print #1,
    For k = 1 To 8
      Print #1, "For x ("; k; ") ="; x(k); ", mass fraction above-size ="; wtfr(k)
      Print #1, "Number fraction above-size ="; q(k)
      If cr(k) = 0 Then
        Print #1, "No. of fragments = 0"
      Else
        Print #1, "No. of fragments ="; cr(k); ", Ln = "; Log(cr(k))
      End If
    Next
    Print #1,

      For k = 1 To 8
        If cr(k) = 0 Then
          GoTo 1
        Else
          count = count + 1
          xm = xm + Log(x(k))
          xsqrt = xsqrt + Log(x(k)) ^ 2
          ym = ym + Log(cr(k))
          xy = xy + Log(x(k)) * Log(cr(k))
        End If
    1 Next
      xmean = xm / count
```

```
    ymean = ym / count
    bottom = xsqrt - count * xmean ^ 2
    top = xy - count * xmean * ymean
    slope = top / bottom
    Print #1, "Slope ="; slope

End Sub
```

Sample of results (for impact energy 1E-4J and normally distributed shape factor)

```
Substances=MS
Vicker Hardness=649000000 Pa
Quasi-isotropic Shear Modulus=9060000000 Pa
Critical Work to form cracks=.0000000048 J
Impact Energy=.0001 J
Ratio of Efficiency Constant=.5
Total number of fragments for 50 samples =1080871
Total volume removed by attrition per sample =2.93957E-11 m3
Lmin =1.998301E-06 rmin =7.36170880796748E-05
Lmax =9.721006E-05 rmax =1.9442012384607E-04

For x ( 1 ) = 1.998301E-06 , mass fraction above-size = 15.94178
Number fraction above-size = 73.40636
No. of fragments = 793428 , Ln =  13.5841180775917
For x ( 2 ) = .000003 , mass fraction above-size = 17.75184
Number fraction above-size = 21.55835
No. of fragments = 233018 , Ln =  12.3588709827828
For x ( 3 ) = .000005 , mass fraction above-size = 20.66718
Number fraction above-size = 4.507013
No. of fragments = 48715 , Ln =  10.7937422698594
For x ( 4 ) = .00001 , mass fraction above-size = 10.7206
Number fraction above-size = .3918136
No. of fragments = 4235 , Ln =  8.35113860708615
For x ( 5 ) = .000015 , mass fraction above-size = 6.506121
Number fraction above-size = 8.261856E-02
No. of fragments = 893 , Ln =  6.7945865808765
For x ( 6 ) = .00002 , mass fraction above-size = 4.964769
Number fraction above-size = 2.895813E-02
No. of fragments = 313 , Ln =  5.74620319054015
For x ( 7 ) = .000025 , mass fraction above-size = 3.83447
Number fraction above-size = .0118423
No. of fragments = 128 , Ln =  4.85203026391962
For x ( 8 ) = .00003 , mass fraction above-size = 4.977971
Number fraction above-size = 8.141582E-03
No. of fragments = 88 , Ln =  4.47733681447821
For x ( 9 ) = .00004 , mass fraction above-size = 7.465596
Number fraction above-size = 4.070791E-03
No. of fragments = 44 , Ln =  3.78418963391826
For x ( 10 ) = .00006 , mass fraction above-size = 7.169665
Number fraction above-size = 8.326618E-04
No. of fragments = 9 , Ln =  2.19722457733622
```

Slope =-3.39262492684397

Substances = PA

Declaration

```
Dim x(1 To 9) As Single
Dim cr(1 To 8), q(1 To 8) As Single
Dim wt(1 To 8), wtfr(1 To 8) As Single
Dim substances, Hv, m, Wc, Wp, ratio As Single
Dim i, j As Single
Dim FracR, N As Single
Dim Amin, Amak, Aa, Ak, A, ran1 As Single
Dim Lmin, Lmak, L, rmin, rmak, r, slope, cons, konst, Va As Single
Dim crysno, cryswt As Single
Dim xm, ym, xy, xmean, xsqrt, ymean, top, bottom As Single
Dim k, count As Integer
```

Main Program

```
Sub main ()

substances = (mersmann.Text1.Text)
Hv = Val(mersmann.Text2.Text) * 1000000
m = Val(mersmann.Text3.Text) * 1000000000
Wc = Val(mersmann.Text4.Text) * .0000000001
Wp = Val(mersmann.Text5.Text) * .000001
ratio = Val(mersmann.Text6.Text)


Open "mer" For Output As #1
Print #1, "Substances=" & substances
Print #1, "Vicker Hardness=" & Hv; " Pa"
Print #1, "Quasi-isotropic Shear Modulus=" & m; " Pa"
Print #1, "Critical Work to form cracks=" & Wc; " J"
Print #1, "Impact Energy=" & Wp; " J"
Print #1, "Ratio of Efficiency Constant=" & ratio

Amin = .5236
Amak = 1
Aa = (Amak + Amin) / 2
Ab = (Amak - Amin) / 6
```

```
FracR = (ratio * Wc ^ (1 / 3) * Hv ^ (5 / 3)) / (5.2 * m)
Lmin = (32 * m * FracR) / (3 * Hv ^ 2)
rmin = (((Lmin * Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (3 * m * FracR)) ^ (1 / 4))
rmax = ((Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (2 * 3 * m * FracR)) ^ (1 / 3)
Lmak = rmax / 2
konst = ((2 / 3) * (m ^ 3) * (FracR ^ 3)) / ((.0007 ^ (4 / 3)) * (Hv ^ 6))
cons = 13 * Log(rmin)
x(1) = Lmin
x(9) = Lmak
Rem For i = 2 To 9
Rem    x(i) = x(i - 1) + (Lmak - Lmin) / 9
Rem Next
x(2) = .000003
x(3) = .000005
x(4) = .00001
x(5) = .000015
x(6) = .00002
x(7) = .000025
x(8) = .00003

For i = 1 To 50
  Debug.Print i
  Randomize
  Ak = (-2 * (Ab ^ 2) * Log(Rnd)) ^ .5
  rand1 = Rnd
  If rand1 < .5 Then
    A = Aa - Ak
  Else
    A = Aa + Ak
  End If
  Rem A = .5236
  N = (.0007 * Wp * (Hv ^ 5)) / (A * m ^ 3 * FracR ^ 3)
  Debug.Print N
  For j = 1 To N
    Randomize
200   r = Exp((cons - Log(Rnd)) / 13)
    L = (3 * m * FracR * r ^ 4) / (Wp ^ (4 / 3) * Hv ^ (2 / 3))
    If L > x(1) Then
      GoTo 250
    Else
      GoTo 200
    End If
250   If L > x(2) Then
      GoTo 260
    Else
      k = 1
```

```
        GoTo 1000
      End If
260   If L > x(3) Then
        GoTo 270
      Else
        k = 2
        GoTo 1000
      End If
270   If L > x(4) Then
        GoTo 280
      Else
        k = 3
        GoTo 1000
      End If
280   If L > x(5) Then
        GoTo 290
      Else
        k = 4
        GoTo 1000
      End If
290   If L > x(6) Then
        GoTo 300
      Else
        k = 5
        GoTo 1000
      End If
300   If L > x(7) Then
        GoTo 310
      Else
        k = 6
      GoTo 1000
      End If
310   If L > x(8) Then
        GoTo 320
      Else
        k = 7
        GoTo 1000
      End If
320   If L > x(9) Then
        GoTo 200
      Else
        k = 8
        GoTo 1000
      End If

1000  cr(k) = cr(k) + 1
```

```
      wt(k) = wt(k) + L ^ 3
    Next
    Va = Va + konst * (N * A) ^ (4 / 3)
Next

For k = 1 To 8
    crysno = crysno + cr(k)
    cryswt = cryswt + wt(k)
Next

For k = 1 To 8
    q(k) = (cr(k) / crysno) * 100
    wtfr(k) = (wt(k) / cryswt) * 100
Next

Print #1, "Total number of fragments for 50 samples =" & crysno
Print #1, "Total volume removed by attrition per sample =" & Va / 50; " m3"
Print #1, "Lmin =" & x(1); " rmin =" & rmin
Print #1, "Lmax =" & x(9); " rmax =" & rmax
Print #1,
For k = 1 To 8
    Print #1, "For x ("; k; ") ="; x(k); ", mass fraction above-size ="; wtfr(k)
    Print #1, "Number fraction above-size ="; q(k)
    If cr(k) = 0 Then
        Print #1, "No. of fragments = 0"
    Else
        Print #1, "No. of fragments ="; cr(k); ", Ln = "; Log(cr(k))
    End If
Next
Print #1,

    For k = 1 To 8
        If cr(k) = 0 Then
            GoTo 1
        Else
            count = count + 1
            xm = xm + Log(x(k))
            xsqrt = xsqrt + Log(x(k)) ^ 2
            ym = ym + Log(cr(k))
            xy = xy + Log(x(k)) * Log(cr(k))
        End If
1   Next
    xmean = xm / count
    ymean = ym / count
    bottom = xsqrt - count * xmean ^ 2
    top = xy - count * xmean * ymean
```

```
slope = top / bottom
Print #1, "Slope ="; slope

End Sub
```

Sample of results (for impact energy 1E-4J and normally distributed shape factor)

```
Substances=PA
Vicker Hardness=754000000 Pa
Quasi-isotropic Shear Modulus=7960000000 Pa
Critical Work to form cracks=.0000000007 J
Impact Energy=.0001 J
Ratio of Efficiency Constant=.5
Total number of fragments for 50 samples =7628530
Total volume removed by attrition per sample =4.80697E-11 m3
Lmin =1.000547E-06 rmin =7.00276821630033E-05
Lmax =1.145268E-04 rmax =2.29053605390034E-04

For x ( 1 ) = 1.000547E-06 , mass fraction above-size = 34.78629
Number fraction above-size = 97.18503
No. of fragments = 7413789 , Ln =  15.8188522026169
For x ( 2 ) = .000003 , mass fraction above-size = 13.17693
Number fraction above-size = 2.279495
No. of fragments = 173892 , Ln =  12.0661896958339
For x ( 3 ) = .000005 , mass fraction above-size = 15.36704
Number fraction above-size = .4786505
No. of fragments = 36514 , Ln =  10.5054510276735
For x ( 4 ) = .00001 , mass fraction above-size = 7.862236
Number fraction above-size = 4.146277E-02
No. of fragments = 3163 , Ln =  8.05927622330565
For x ( 5 ) = .000015 , mass fraction above-size = 5.257358
Number fraction above-size = 9.451362E-03
No. of fragments = 721 , Ln =  6.58063913728495
For x ( 6 ) = .00002 , mass fraction above-size = 3.616263
Number fraction above-size = 3.014998E-03
No. of fragments = 230 , Ln =  5.4380793089232
For x ( 7 ) = .000025 , mass fraction above-size = 3.306294
Number fraction above-size = 1.428847E-03
No. of fragments = 109 , Ln =  4.69134788222914
For x ( 8 ) = .00003 , mass fraction above-size = 4.096673
Number fraction above-size = 9.044993E-04
No. of fragments = 69 , Ln =  4.23410650459726
For x ( 9 ) = .00004 , mass fraction above-size = 5.638319
Number fraction above-size = 4.194779E-04
No. of fragments = 32 , Ln =  3.46573590279973
For x ( 10 ) = .00006 , mass fraction above-size = 4.217582
Number fraction above-size = 1.179782E-04
No. of fragments = 9 , Ln =  2.19722457733622
For x ( 11 ) = .00008 , mass fraction above-size = 2.675012
Number fraction above-size = 2.621737E-05
No. of fragments = 2 , Ln =  .693147180559945

Slope =-3.40685649605822
```

OVERALL

CONCLUSIONS

AND

RECOMMENDATIONS

OVERALL CONCLUSIONS

Mathematical analysis of transient CSD and attrition phenomena is a difficult task. The advantage of the MC simulation lies in the fact that the CSD can be computed in terms of weight fraction of the crystals, which is mathematically difficult by usual moment transformation of population balance. The scheme can also be easily implemented on a personal computer, as the demand on its memory is very small.

In transient CSD analysis, the simulation scheme is attractive due to its simple algorithm, i.e. free from iterative calculations and related convergence problems. The simulated transient CSD can account for size-dependent growth. It could also incorporate improper mixing conditions under all possible dispersion effects. The agreement among the predicted results and available experimental data confirm the validity of the proposed scheme.

In the attrition process, MC offers an alternative to the conventional deterministic approach in solving the Random Breakage model, described by a second order ODE, as proposed by RANDOLPH (1969). In addition, a comparison between experimental CSD results under attrition conditions with the MC simulated CSD, which employed the model proposed by MAZZAROTTA (1992) and BISCANS *et al.* (1996), indicate good agreement. To further enhance its credibility, MC methods was also successfully incorporated into GAHN and MERSMANN (1997) model for generating

the fragment size distribution resulted from attrition. The simulated results account for volume shape factor dispersion which GAHN and MERSMANN (1997) assumed a value of 1.

From the practical aspects, the work done has contributed to the design of industrial crystallizer that is still teeming with empiricism due to lack of reliable knowledge on the crystal size distribution. Industrial and commercial products such as sugar, common salt, urea and other fertilizers, which are marketed in the crystal form, have to be produced now in various size ranges to meet varied consumer demand. Detailed knowledge on crystal size distribution and the factors that influencing it will allow the engineers to design and operate crystallizers successfully and economically. As it is not always possible to perform lab or pilot-scale studies to test out a chosen configuration, simulation schemes such as the present work are used. The economy and time factor in such scheme make them attractive to designers and operators. The findings in the present work show that MC technique in association with the appropriate physical and mathematical models can be used to predict the CSD reliably for various crystallizer configurations and operating conditions. The results indicate that the MC technique simulates closely the randomness of the crystallization process.

RECOMMENDATIONS

Some of the recommendations that could be thought off to ensure the continuity of the current work are outlined below:

1. More experimental work needs to be carried out as there are limited published data being reported in the field of crystallization. The experimental work should emphasized on the transient CSD in which the MC simulation results could be tested and compared against the actual transient data.

2. In addition to attrition, the effect of agglomeration on the CSD should be considered in the simulation schemes, as one could not discount it in the actual crystallization processes.

3. The sensitivity of solubility on the CSD should be explored and incorporated into the MC simulation schemes.

NOMENCLATURE

NOMENCLATURE

A'	growth constant (m/s)
a	constant
a'	constant
a''	defined in Eq. (1.139)
a_1	defined in Eq. (1.124)
a	characteristic size of the plastic zone (m)
a_p	liquid-solid contact area of a particle (m^2/m^3)
B	break terms representing crystal breakage
B'	constant
b	ASL parameter
C	constant
c	concentration, kg solute/kg solution
	constant, [Chap. 7]
c^*	equilibrium concentration, kg solute/kg solution
c_1, c_2, c_3	constant
D	diameter of a crystal (m, mm)
	death term representing crystal breakage, [Chap 5]
D_G	growth rate diffusivity (m^2/s)
D_i	diffusivity (m^2/s)
D_f	fluid phase diffusivity (m^2/s)

D_i	diameter of the i th. crystal (m, mm)
D_{ik}	diameter of the i th. crystal, measured after the k th. crystallizer (m, mm)
D_{ikl}	diameter of the i th. crystal, formed by nucleation in the l th. crystallizer and measured after the k th. crystallizer (m, mm)
E	step operator
E	mean
$F(S)$	function of supersaturation in Eqs (1.96), (1.97) and (1.100).
F_1, F_2	growth rate parameters defined in Eqs. (1.25) and (1.26)
$f_1(\phi), f_2(\phi), f_3(\phi)$	function defined in Eqs. (AI.6.3), (AI.6.7) and (AI.6.8) respectively
$f(g_i), f(G_i)$	function defined by Eqs. (1.28) and (1.29)
$f(j)$	function defined by Eqs. (1.112) and (1.113)
$f(x)$	density function as defined by Eqs. (AI.3.1)
f_1, f_2	defined in Eqs. (1.20) and (1.21)
f_{ij}	defined in Eq. (1.78)
$\overline{f_i}$	defined in Eq. (1.79)
$G(L)$	function of crystal length in Eq. (1.96)
G	growth rate of a crystal (m/s, mm/hr)
	linear crystal growth rate, [Chap. 5] (mm/min)
$\overline{G}$	average growth rate of a crystal (m/s, mm/hr)
G_M	maximum growth rate (m/s, mm/hr, mm/min)
G_{Mk}	maximum growth rate of a crystal in the k th. crystallizer (m/s, mm/hr)

G_O	minimum growth rate (m/s, mm/hr, mm/min)
G_{Ok}	minimum growth rate of a crystal in the k th. crystallizer (m/s, mm/hr)
G_i	growth rate of the i th. crystal (m/s, mm/hr, mm/min)
G_{ik}	growth rate of the i th. crystal in the k th. crystallizer (m/s, mm/hr)
G_o	growth rate of crystals at zero size (m/s)
g_i	instantaneous growth rate of the i th. crystal (m/s)
$H(j)$	function of states j in Eqs (1.96), (1.101), (1.107) and (1.109)
H	dynamic hardness (Pa)
H_v	Vickers hardness (Pa)
h	step size
I	value of the integral defined in Eq. (1.73)
i	index for a crystal
j	index for mesh size interval
	index for piece [Eqs. (1.74) through (1.79)]
$i, j, k, k_1, k_2, \ldots$	different states in the Markov process [Eqs. (1.82) through (1.90), Eqs(AI.4.1) through AI.4.8)]
K	volume shape factor of a crystal
	as defined in Eq. (1.71)
$\overline{K}$	average volume shape factor of a crystal
K_1, K_2	reaction rate constants
K_M	maximum volume shape factor
K_{Mk}	maximum volume shape factor in the k th. crystallizer

K_O	minimum volume shape factor
K_{Ok}	minimum volume shape factor in the k th. crystallizer
K_i	volume shape factor of the i th. crystal
K_{ik}	volume shape factor of the i th. crystal in the k th. crystallizer
K_r	efficiency of stress field created by the crystal-impeller impact
k	constant
k	number of crystallizers in series
k'	constant
k^*	as defined in Eq. (1.1)
k_1, k_2, k_3, k_4	reaction rate constants [Eqs. (1.97) through (1.101), Eqs (1.108), (1.110) and (1.111)]
k_b	fraction of attrition given by breakage mechanism
k_c	fraction of attrition given by abrasion mechanism generating coarse fragments
k_d	volume diffusion rate constant (m/s)
k_f	fraction of attrition given by abrasion mechanism generating fine fragments
k_n	no. of pieces in a given size range
L	birth size of a crystal (m)
	crystal size, [Eqs. (1.68), (1.70), (1.72) through (1.75), (1.96), (1.107), (1.109), Chap. 2 and 6] (m)
	linear crystal dimension, [Chap. 5] (mm)

	fragment size, [Chap. 7] (m)
L^*	characteristic size for Broadbent-Callcott expressions, corresponding to maximum size distribution under examination (μm)
L'	characteristic size of a crystal (m)
L_M	mean (arithmetic mean) birth size of a crystal (mm)
L_i	birth size of the i th. crystal (mm, m)
	size of the i th. crystal, [Eq. (1.80), Chap. 2] (m)
	linear crystal dimension of the i th. crystal, [Chap. 5] (mm)
L_{il}	birth size of the i th. crystal in the l th. crystallizer (mm, m)
L_j	characteristic length of a crystal in state j for the Markov process
L_c	minimum size of coarse fragments generated by abrasion (μm)
L_f	maximum size of fine fragments generated by abrasion (μm)
$L_{\max}$	maximum size of a fragment (m)
$L_{\min}$	minimum size of a fragment (m)
l	summation index
l'	number of trials
$\mathrm{M}(t)$	number of available solutes molecules at time t
M	number of available solutes molecules
M	total number of states in the Markov process
m	defined in Eq. (1.125)
$m(0)$	number of seeds generated in the crystallizer initially
m	number of mesh size intervals in which sampled crystals are classified

	mean as defined in Eq. (AI.3.4)
	parameter indicative of the breakage rate, [Chap. 5]
	mean volume shape factor, [Chap. 7]
m_j	sample size in the j th. piece
$N_j(t)$	number of crystals in state j at time t [Eqs. (1.105) and (1.106), Appendix I.5]
N	number of crystallization stages
	total number of fragments, [Chap. 7]
N'	number of fluid elements in a CSTR
N_M	maximum nucleation rate (1/min)
N_O	minimum nucleation rate (1/min)
N_T	nucleation rate at time T (1/min)
$n(L)$	crystal population density at crystal size L (no./kg.m)
$n(L')$	crystal population density at crystal size L' (no./kg.m)
n	sample size of the crystals
	n -steps in the Markov process [Eqs. (1.82 through (1.90)]
	population density, [Chap. 2 and 5] ($1/m^4$, $1/mm^4$)
	distribution modulus, [Chap. 6]
	number of crystals in the Markov process [Appendix I.5]
n'	size population density (no./kg.m)
n^l	number of crystals in sample, formed by nucleation in the l th. crystallizer
n^o	nuclei density ($1/mm^4$)

n_j^l	number of crystals formed by nucleation in the lth. crystallizer and grown into the j th. size interval
$n_1, n_2, \ldots n_m$	number of crystals in 1, 2, ...m mesh size intervals
$n_1, n_2, \ldots n_M$	number of crystals in state 1, 2, ... M in the Markov process [Eqs. (1.102) and (1.103), Appendix I.5]
n_b	modulus of size distribution generated by breakage mechanism
n_c	modulus of size distribution generated by coarse fragments generated by abrasion mechanism
n_f	modulus of size distribution generated by fine fragments generated by abrasion mechanism
n_j	number of crystals in the j th. size interval
n_o	crystal population density at zero size (no./kg.m)
$P_j(t)$ or $P(j,t)$	probability of a crystal to be in state j, $j = 1, 2, \ldots M$ at time t
P'	transition probability matrix
P^n	n-steps transition probability matrix
P_{ij}^n	conditional probability that random variable X, starting from state i will be in state j after n-steps
P_{ij}	conditional probability that random variable X, starting from state i will be in state j after one-step
p	size range
$q_o(L)$	number density distribution (m^{-1})

$q(x,t)$	probability function with one-to-one correspondence to $P(j,t)$
q	fractional breakage at $x = 3$ relative to product removal
RND	random number, range 0 to 1
r	distance from the peak of the cone (m)
$\bar{r}$	mean pore radius
r_{max}	maximum distance from the peak of the cone to the newly created surface (m)
r_{min}	minimum pore radius
S	dummy variable of integration
	standard error [Eq. (1.77)]
	supersaturation [Eqs. (1.96), (1.97) and (1.100)]
s	standard deviation for volume shape factor
T	non-negative integers
T	residence time of a crystal in a crystallizer (hr, min, s)
T_i	residence time of the i th. crystal (hr, min, s)
T_{ik}	residence time of the i th. crystal in the k th. crystallizer (hr, min, s)
t	time (hr, min, s)
u	dye concentration in a capillary
v	v-steps in the Markov process
V	total volume of crystallizer (m^3)
V_M	volume at perfect mixing in crystallizer (m^3)
V_P	volume at plug flow in crystallizer (m^3)
V_a	total volume removed by attrition (m^3)

V_{el}	volume removed in the elastic region (m^3)
Var	variance
W	total weight of n crystals (g)
W_k^j	mass fraction of crystals in the j th. size interval measured after the k th. crystallizer
W_{lk}^j	mass fraction of crystals formed by nucleation in the l th. crystallizer, grown into the j th. size interval and measured after the k th. crystallizer
W_c	critical work to form crack (J)
W_i	weight of the i th. crystal (g)
W_{ik}	weight of the i th. crystal in the k th. crystallization unit (kg)
W_p	impact energy (J)
w	cumulative weight distributions (g)
$\dot{w}$	differential weight distributions (g)
$\ddot{w}$	second order differential equation for cumulative weight distributions (g)
w_p	cumulative mass fraction of crystals in the p th. size range
w_T	total weight of a sample size of crystal (g)
WF_j	weight fraction of the crystals in the j th. size interval
WC_j	cumulative weight fraction of the crystals up to the j th. size interval
X	random variable
$X_1, X_2, \ldots, X_M$	random vector

X_t or $X(t)$	random variable in the Markov process at time t
x	dimensionless size
	realization of the random variable, X, [Eqs. (1.112) through (1.143), (AI.6.1) through (AI.6.8)]
	dimensionless size, L/L^*, [Chap. 6]
$\langle x \rangle$	mean of the random variable X
$\langle x^2 \rangle$	second moment of the random variable X
$x_1, x_2, \ldots, x_M$	probability mass fraction, [Appendix I.5]
y	dimensionless population density
	defined in Eq. (1.126). It represents the ratio between the fluctuating and macroscopic components of j as indicated in Eq. (1.117).
$\dot{y}$	$\frac{dy}{dx}$
y_w	overall cumulative size distribution of fragments generated by attrition
$y_{w,b}$	cumulative size distribution of fragments generated by breakage mechanism
$y_{w,c}$	cumulative size distribution of coarse fragments generated by abrasion mechanism
$y_{w,f}$	cumulative size distribution of fine fragments generated by abrasion mechanism

GREEK LETTERS

α	volume shape factor
$\alpha_1, \alpha_2, \ldots \alpha_{k_n}$	crystal size in the 1, 2,...k_n piece
α_{max}	maximum volume shape factor
α_{min}	minimum volume shape factor
α_n	normally distributed volume shape factor, ranges from α_{min} to α_{max}
β_m	dimensionless parameter indicative of the importance of breakage in the system
δ	segment length in each state for the Markov process
$\phi(t)$	deterministic function of t representing the macroscopic trajectory of crystal size which gives rise to a sharp maximum in the value of $P(j,t)$ around the $\Omega\phi(t)$ [Eqs. (1.112) through (1.143), Appendix I.6]
ϕ	polar coordinates (rad)
γ	as defined in Eq. (1.69) (m^{-1})
$\gamma_0, \gamma_1, \gamma_2, \cdots \gamma_k$	random numbers
κ	parameter κ =0.5
λ	transition intensities among the states in the Markov process
$\lambda_j(t)$	transition intensity (probability of migration) in state j at time t used in the Markov process
$\lambda(L)$	initial size population density function

λ_1	scale parameter for the initial size population density function
λ_2	shape parameter for the initial size population density function
μ	shear modulus (Pa)
μ_{VRH}	quasi-isotropic shear modulus (Pa)
$o(\Delta t)$	higher-order term of the transition intensities, λ
θ	polar coordinates (rad)
θ'	mean residence time of crystals (s)
$\theta^*(T)$	residence time distribution function
ρ	crystal density (g/m^3, g/mm^3, kg/mm^3)
σ	relative supersaturation, $(c - c^*)/c^*$
	mean square deviation of growth rate [Eqs. (1.29), (1.32) and (1.33)] (m^2/s^2)
	stress, [Chap. 7] (Pa)
	standard deviation [Eq. (AI.3.5)]
σ'	relative supersaturation, $(c - c^*)/c^*$
σ_i	mean square deviation of growth rate of the i th. crystal (m^2/s^2)
Γ	fracture resistance (Jm^{-2})
τ	mean residence time of crystals in a crystallizer (hr, min, s)
	residence time in an imperfectly mixed crystallizer, [Chap. 3 and 4] (hr, min)
$\overline{\tau_N}$	mean residence time of crystals in the N th. stage (min)
τ_M	mean residence time of crystals in perfectly mixed region (min)

τ_p	mean residence time of crystals in plug flow region (min)
τ_k	mean residence time of crystals in the k th. crystallizer (hr, s)
$\upsilon(G)$	growth rate population density function
υ_1	shape parameter for the growth rate population density function
$\xi_{i,j}$	random number generated for the i th. crystal in the j th. piece random number [Eqs. (1.76) and (1.78)]
$\xi_{i,N}$	random number generated for the i th. crystal in the N th. stage
Ω	maximum obtainable crystal size

FLOW CHART

NOTATION

FLOW CHART NOTATION

a	characteristic size of the plastic zone (m)
Beta	dimensionless parameter indicative of the importance of breakage in the system
CRYSNO (T, SIZE)	counts the number of crystals in a particular size class at time T
CRYSWT(T, SIZE)	records the mass of crystals in a particular size class at time T (g)
cr(J)	number of fragments in the J th. size classes
cr(k)	count the number of crystals in size class k before birth and death function consideration
cr_2(k)	count the number of crystals in size class k after birth and death function consideration
cryswt	total weight of sample crystals before birth and death consideration (g)
cryswt_2	total weight of sample crystals after birth and death consideration (g)
DI(T)	diameter of the *i* th. crystal at time T (mm)
D(k)	number of crystals death in size class k
Di	diameter of the *i* th. crystal (mm)
F1	growth rate parameter defined in Eq. (1.25) for F_1 (mm^2/min)
F2	growth rate parameter defined in Eq. (1.26) for F_2 ($mm^{2.5}$/min)
G	average growth rate of a crystal, [Appendix III.1 and IV.1] (mm/hr); growth rate of crystal, [Appendix V.2] (mm/min)
GI	growth rate of the *i* th. crystal (mm/min)
GM	maximum growth rate (mm/min)

GO	minimum growth rate (mm/min)
I	counter for the number of crystals, [Appendix V.1];
	counter for the number of fragments, [Appendix VII.1]
J	particle size classes
K	shape factor of crystal
KI	shape factor of the *i* th. crystal
KM	maximum shape factor
KO	minimum shape factor
K_r	efficiency of stress field created by the crystal-impeller impact
k	number of size classes or mesh
L	fragment size (m)
LI	birth size of the *i* th. crystal (mm)
LM	mean (arithmetic mean) birth size of a crystal) (mm)
L(k)	linear crystal dimension in size class k (mm)
Lmin	minimum size of a fragment (m)
Lmak	maximum size of a fragment (m)
m	parameter indicative of the breakage rate
N	number of stages for DTB crystallizers, [Appendix III.1 and IV.1]
N	total number of fragments, [Appendix VII.1]
NCOUNT	nucleation counter, range 1 to NRATE
NM	maximum nucleation rate (nuclei/min)
NO	minimum nucleation rate (nuclei/min)
NRATE	nucleation rate (nuclei/min)

p	crystal density (g/mm^3)
q	fractional breakage at $x = 3$ relative to product removal
R	ratio of plug flow regions to total volume of crystallizer for FC crystallizers
RND	random number, range 0 to 1
r	distance from the peak of the cone (m)
rmak	maximum distance from the peak of the cone to the newly created surface (m)
SIZE	size-classes, [Appendix III.1 and IV.1] (mm)
sample	sample size of crystal in a steady state run, [Appendix I.1 and V.1]; number of crystals used in simulation, [Appendix VII.1]
size	diameter of the i th. crystal, [Appendix V.1] (mm)
TAU	average residence time of crystals in a crystallizer (hour, min)
TCOUNT	time counter, range 1 min to TSCAN (min)
TI	residence time of the i th. crystal on a continuous time scale, range TCOUNT to TTOT. (TI = -TAU.LOG(RND)), [Appendix III.1 and IV.1] (min)
TM	mean residence time of crystals in perfectly mixed region (min)
TP	mean residence time of crystals in plug flow region (min)
TSCAN	predetermined search time for observation of CSD in a continuous crystallizer. It is arbitrarily fixed at 13 times the average residence time of crystals in the crystallizer (TSCAN = 13.TAU) (min)

TTOT	exit time of a crystal on a continuous time scale. If TTOT is greater than TSCAN, it is fixed at TSCAN, otherwise, TTOT = TCOUNT + TI (min)
Ti	residence time of the *i* th. crystal, [Appendix I.1 and V.1] (min)
WT	total weight of crystals (g)
W(J)	weight of crystals in the J th. size classes (g)
WF(J)	weight fraction of the crystals in the J th. size classes
WI(T)	mass of the *i* th. crystal at time T (g)
Wp	impact energy (J)
w(J)	weight of fragment in the J th. size classes (g)
wf(J)	weight fraction of the fragments in the J th. size classes
wf(k)	weight fraction of the sample crystals in size class k before birth and death consideration
wf_2(k)	weight fraction of the sample crystals in size class k after birth and death consideration
wt	total weight of fragments (g)
wt(k)	total weight of sample crystals in size class k before birth and death consideration (g)
wt_2(k)	total weight of sample crystals in size class k after birth and death consideration (g)
x(k)	dimensionless length of crystals in size class k
α	volume shape factor
Γ	fracture resistance (Jm^{-2})